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Algebraic structures of soft sets associated with new operations

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ABSTRACT

Recently new operations have been defined for soft sets. In this paper, we study some important properties associated with these new operations. A collection of all soft sets with respect to new operations give rise to four idempotent monoids. Then with the help of these monoids we can study semiring (hemiring) structures of soft sets. Some of these semirings (hemirings) are actually lattices. Finally, we show that soft sets with a fixed set of parameters are MV algebras and BCK algebras.

0. Introduction

Primarily the aim of soft set theory is to provide a tool with enough parameters to deal with uncertainty associated with the data, whereas on the other hand it has the ability to represent the data in a useful manner. With the introduction of the so-called new operations in soft sets, it is imperative to study the underlying algebraic structures. This will give a forehand and better understanding for their applications. For different applications of soft sets see [1–8].

In the study of soft sets as algebraic structures there are mainly two types of collections of soft sets. First the collection of soft sets with a fixed set of parameters, and second the collection of soft sets with different sets of parameters. These two types of collections with new operations sometimes behave similarly and sometimes differently.

On soft sets different binary operations are defined and a general formula for this purpose is available [9]. In this study we restrict ourselves to those operations which are given in [10]. Although soft sets over algebraic structures have been studied extensively [11–15] yet the algebras of soft sets itself, did not get so much attention. Initially lattice of soft ideals of a soft semigroup was studied in [14]. Recently Qin and Hong [16] have studied lattices of soft sets with respect to new operations. However as long as we know, no systematic study of algebraic structures associated with new operations has been done yet.

There are many algebras associated with logic. Boolean algebras are associated with traditional two valued Aristotelean logic. MV algebras are suitable for multi-valued logic. BCI/BCK algebras generalize the notion of algebra of sets with the set subtraction as the only non-nullary operation and on the other hand, these algebras generalize implication algebras. In this paper, we study algebraic structures of soft sets associated with the new operations in a systematic way.

This paper is arranged in the following manner. In Section 1, some definitions and notions about soft sets and algebraic structures such as semigroups, semirings and lattices are given. These definitions will help us in later sections. Section 2, completely describes for what binary operations distributive laws hold. In Section 3, monoids, semirings and lattices of soft

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sets associated with new operations have been determined completely. Finally in Section 4, algebraic structures of soft sets with a fixed set of parameters associated with new operations are studied. It is seen that this collection becomes Stone's algebra and MV algebra. On the other hand this collection becomes a BCK algebra with respect to restricted difference.

1. Preliminaries

In this section, some definitions and notions about soft sets and algebraic structures are given. These will be useful in later sections.

Let U be an initial universe and E be a set of parameters. Let $\mathcal{P}(U)$ denote the power set of U and A, B be non-empty subsets of E.

Definition 1 (*[17]*)**.** A pair (F, A) is called a *soft set* over U, where F is a mapping given by $F : A \to \mathcal{P}(U)$.

In other words, a soft set over U is a parameterized family of subsets of the universe U. For $e \in A$, $F(e)$ may be considered as the set of e-approximate elements of the soft set (F, A).

Definition 2 (*[18]*)**.** For two soft sets (F, A) and (G, B) over a common universe U, we say that (F, A) is a *soft subset* of (G, B) if

(1) $A \subseteq B$ and
(2) $F(e) \subseteq G(e)$ for all $e \in A$.

We write $(F, A)\widetilde{\subseteq}(G, B)$.
In this case (G, B) is said to be a soft super set of (F, A).

Definition 3 (*[18]*)**.** Two soft sets (F, A) and (G, B) over a common universe U are said to be *soft equal* if (F, A) is a soft subset of (G, B) and (G, B) is a soft subset of (F, A).

Definition 4 (*[10]*)**.** Let U be an initial universe, E be the set of parameters, and $A \subseteq E$.

(a) (F, A) is called a relative null soft set (with respect to the parameter set A), denoted by $\emptyset_A$, if $F(a) = \emptyset$ for all $a \in A$.
(b) (G, A) is called a relative whole soft set (with respect to the parameter set A), denoted by $\mathfrak{U}_A$, if $G(e) = U$ for all $e \in A$.

The relative whole soft set with respect to the set of parameters E is called the *absolute soft set* over U and simply denoted by $\mathfrak{U}_E$. In a similar way, the relative null soft set with respect to E is called the *null soft set* over U and is denoted by $\emptyset_E$.

We shall denote by $\emptyset_\emptyset$ the unique soft set over U with an empty parameter set, which is called the *empty soft set* over U. Note that $\emptyset_\emptyset$ and $\emptyset_A$ are different soft sets over U and $\emptyset_\emptyset \subseteq \emptyset_A \subseteq (F, A) \subseteq \mathfrak{U}_A \subseteq \mathfrak{U}_E$ for all soft set (F, A) over U.

Definition 5 (*[10]*)**.**

(1) Extended *union of two soft sets* (F, A) and (G, B) over the common universe U is the soft set (H, C), where $C = A \cup B$ and for all $e \in C$,

$$H(e) = \begin{cases} F(e) & \text{if } e \in A - B \\ G(e) & \text{if } e \in B - A \\ F(e) \cup G(e) & \text{if } e \in A \cap B. \end{cases}$$

We write $(F, A) \cup_{\mathcal{E}} (G, B) = (H, C)$.

(2) Let (F, A) and (G, B) be two soft sets over the same universe U, such that $A \cap B \neq \emptyset$. The *restricted union* of (F, A) and (G, B) is denoted by $(F, A) \cup_{\mathcal{R}} (G, B)$ and is defined as $(F, A) \cup_{\mathcal{R}} (G, B) = (H, C)$, where $C = A \cap B$ and for all $e \in C$, $H(e) = F(e) \cup G(e)$.

If $A \cap B = \emptyset$, then $(F, A) \cup_{\mathcal{R}} (G, B) = \emptyset_\emptyset$.

Definition 6 (*[10]*)**.**

(1) *The extended intersection* of two soft sets (F, A) and (G, B) over a common universe U, is the soft set (H, C) where $C = A \cup B$ and for all $e \in C$,

$$H(e) = \begin{cases} F(e) & \text{if } e \in A - B \\ G(e) & \text{if } e \in B - A \\ F(e) \cap G(e) & \text{if } e \in A \cap B. \end{cases}$$

We write $(F, A) \cap_{\mathcal{E}} (G, B) = (H, C)$.

(2) Let (F, A) and (G, B) be two soft sets over the same universe U such that $A \cap B \neq \emptyset$. The *restricted intersection* of (F, A) and (G, B) is denoted by $(F, A) \cap_{\mathcal{R}} (G, B)$ and is defined as $(F, A) \cap_{\mathcal{R}} (G, B) = (H, A \cap B)$ where $H(e) = F(e) \cap G(e)$ for all $e \in A \cap B$.