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Least-squares modification of extended Stokes' formula and its second-order radial derivative for validation of satellite gravity gradiometry data

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ABSTRACT

The gravity anomalies at sea level can be used to validate the satellite gravity gradiometry data. Validation of such a data is important prior to downward continuation because of amplification of the data errors through this process. In this paper the second-order radial derivative of the extended Stokes' formula is employed and the emphasis is on least-squares modification of this formula to generate the second-order radial gradient at satellite level. Two methods in this respect are proposed: (a) modifying the second-order radial derivative of extended Stokes' formula directly, and (b) modifying extended Stokes' formula prior to taking the second-order radial derivative. Numerical studies show that the former method works well but the latter is very sensitive to the proper choice of the cap size of integration and degree of modification.

1. Introduction

Satellite gravity gradiometry (SGG) is a technique to measure second-order derivatives of the Earth's gravity field from space. It is expected to determine the geopotential coefficients to higher degrees and orders than those are obtained from other satellite techniques. The SGG data can be used to study the geophysical/geodynamical phenomena as well. Quality of the data is important, as occurrence of any error in the data will lead to a wrong interpretation and unrealistic conclusions for the phenomena. Therefore, the quality of SGG data should be controlled prior to use, or in other words, the data should be validated.

Different methods of validating SGG data have been proposed. A simple way could be the direct comparison of the real SGG data with the synthesized gravitational gradients using an existing Earth's gravitational model (EGM). Another idea is to use regional gravity data to generate the gradients at satellite level. Haagmans et al. (2002) and Kern and Haagmans (2004) used the extended Stokes formula (ESF) and extended Hotine formula to generate the gravitational gradients using terrestrial gravity data. Denker (2002) used the least-squares spectral combination technique to generate and validate the gravitational gradients. Bouman et al. (2003) have set up a calibration model based on instrument (gradiometer) characteristics to validate the measurements. Mueller et al. (2004) used the terrestrial gravity anomalies to generate the gravitational gradients, and after that Wolf (2007) investigated the deterministic approaches to modify the integrals and validate the SGG data. In fact, the spectral weighting scheme (Sjöberg, 1980, 1981; Wenzel, 1981) was used by Wolf (2007). Stochastic methods of modifying Stokes' formula, or in other words leastsquares modification (LSM) can be used for the extended Stokes formula as well; see Sjöberg (1984a,b, 1991, 2003). Least-squares collocation can be used for validation purposes. Tscherning et al. (2006) considered this method and concluded that the gradients can be predicted with an error of 2–3 mE in the case of an optimal size of the collection area and optimal resolution of data. Zielinsky and Petrovskaya (2003) proposed a balloon-borne gradiometer to fly at 20–40 km altitude simultaneously with satellite mission and proposed downward continuation of satellite data and comparing them with balloon-borne data. Bouman and Koop (2003) presented an along-track interpolation method to detect the outliers. Their idea is to compare the along-track interpolated gradients with measured gradients. If the interpolation error is small enough the differences should be predicted reasonably by an error model. Pail (2003) proposed a combined adjustment method supporting high quality gravity field information within the well-surveyed test area for continuation of local gravity field upward and validating the SGG data.

Abbreviations: BLSM, biased least-squares modification; EGM, Earth's gravitational model; ESF, extended Stokes' function; ESK, extended Stokes' kernel; LSM, least-squares modification; OLSM, optimum least-squares modification; RMSE, root mean square error; SGG, satellite gravity gradiometry; SORD, second-order radial derivative; ULSM, unbiased least-squares modification.

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Bouman et al. (2004) stated that there are some limitations in generating the gravitational gradients using terrestrial gravimetry data and EGMs. When an EGM model is used, high degrees and orders should be taken into account and the recent EGMs seem to be able to remove the greater part of the systematic errors. In their regional approach they concluded that the bias of the gradients can accurately be recovered using least-squares collocation. Also, they concluded that the method of validation using high–low satellite-to-satellite tracking data fails unless a higher resolution EGM is available. Kern and Haagmans (2004) and Kern et al. (2005) presented an algorithm for detecting the outliers in the SGG data in the time domain.

The second-order radial derivative (SORD) of extended Stokes' kernel (ESK) is isotropic and azimuth-independent. The isotropy is an important property in modifying ESF otherwise it will not be an easy task. Two methods of generating the SGG data are investigated in this paper, in the first method (Method 1), the SORD of ESF is modified (derivative prior to modification) and in the second method (Method 2) ESF is modified and after that the SORD is taken (modification prior to derivative). Modification of ESF and its SORD based on the biased LSM (BLSM), unbiased LSM (ULSM) and optimum LSM (OLSM) is the main subject of this study which is a new issue in the scope of SGG. Obviously, Methods 1 and 2 will not deliver the same results, but we are going to test in which cases these methods are comparable. We select the SORD of ESF as its kernel function is isotropic, in such a case, we can use both methods to generate the second-order radial gradient and compare the results. The importance of this study is mostly related to Method 2 although Method 1 (based on the LSM) is new as well. If we can find the cases, where Method 2 performs well, the method can be used to some how modify the horizontal derivatives of ESF having non-isotropic kernels, to generate the other gradients. A similar study was done by Wolf (2007) but just based on deterministic approaches. However we concentrate on generating the second-order radial gradient based on the LSM approaches.

The disturbing potential can be expressed by an integral which is well known as ESF. This integral formula is (Heiskanen and Moritz, 1967):

$$T(P) = \frac{R}{4\pi} \iint_{\sigma} S(r,\psi)\Delta g(Q)d\sigma, \tag{1a}$$

where R is the radius of the reference sphere, r is the geocentric distance at computation point P, ψ is the geocentric angle between the computation point P and the integration point Q with the following expression:

$$\cos\psi = \cos\theta\cos\theta' + \sin\theta\sin\theta'\cos(\lambda' - \lambda) \tag{1b}$$

and θ and λ are the co-latitude and longitude of P and θ' and λ' are of the integration point Q. σ is the unit sphere, $\Delta g\,(Q)$ is the gravity anomaly at sea level and

$$S(r,\psi) = \sum_{n=2}^{\infty} \frac{2n+1}{2} \Omega_n(r) P_n(\cos\psi), \tag{1c}$$

is the spectral form of ESK with the spectrum:

$$\Omega_n(r) = \frac{2}{n-1}\left(\frac{R}{r}\right)^{n+1}. \tag{1d}$$

Eq. (1a) shows that the integration should be performed globally, which means that $\Delta g\,(Q)$ with a global coverage is required. Therefore we should look for an approach to modify the integral in such a way that the contribution of the far zone data is minimized. Different methods for modifying Stokes' formula have been presented, but here the concentration is on the stochastic approaches of Sjöberg (1984a,b). In fact, the theory behind this part of the study was presented by him, but just on Stokes' integral for geoid determination. However, we are going to test the capability of these stochastic approaches in modifying ESF and its SORD and generating the SGG data for validation purposes. In the following we investigate the LSM of ESF.

2. LSM of ESF

The general estimator of the disturbing potential based on ESF is very similar to the general geoid estimator of Sjöberg (2003); and the only difference is related to the kernel function and its spectrum. Let us start the discussion by this general disturbing potential estimator:

$$\tilde{T}(P) = \frac{R}{4\pi} \iint_{\sigma_0} S^L(r,\psi)\Delta g^T(Q)d\sigma + \frac{R}{2}\sum_{n=2}^{L} b_n(r)\Delta g_n^{\mathrm{EGM}}(P), \tag{2a}$$

where L is the maximum degree of modification, $b_n(r)$ is a parameter which differs with the type of the LSM, and

$$S^L(r,\psi) = S(r,\psi) - \sum_{n=2}^{L} \frac{2n+1}{2} s_n(r) P_n(\cos\psi), \tag{2b}$$

is the modified ESF and $s_n(r)$ are the modification parameters, which are estimated. The closed form formula of this function is (Heiskanen and Moritz, 1967, p. 93, Eq. 2-162):

$$S(r,\psi) = \frac{2R}{l} + \frac{R}{r} - 3\frac{Rl}{r^2} - \frac{R^2}{r^2}\cos\psi\left(5 + 3\ln\frac{r - R\cos\psi + l}{2r}\right), \tag{2c}$$

where

$$l = \sqrt{r^2 + R^2 - 2Rr\cos\psi}, \tag{2d}$$

is the spatial distance between the points P and Q. $\Delta g_n^{\mathrm{EGM}}(P)$ is the Laplace harmonic expansion of $\Delta g(P)$ (Heiskanen and Moritz, 1967, p. 97). In order to show from which sources the gravity anomaly is derived we separate them into Δg^T for the terrestrial and Δg^{EGM} for the EGM based data.The LSM parameters $s_r(r)$ are derived based on solving the following system of equations (Sjöberg, 2003):

$$\sum_{r=2}^{M} a_{kr} s_r(r) = h_k(r), \quad k = 2, 3, \ldots, M, \tag{2e}$$

where mathematical forms of a_{kr} and $h_k\,(r)$ depend on type of the LSM which is used.

Eq. (2e) differs with the system of equations in which the modification parameters of the Stokes formula is used for geoid determination. As Eq. (2e) shows both sets of the modification parameters and the truncation coefficients are altitude-dependent and variable with the elevation of the computation point P. We will investigate the changes in these parameters and coefficients in Section 4. The mathematical formula of the elements of coefficient matrix and the right hand side vector of Eq. (2e) depend on the method of the LSM. In the following we summarized them in three propositions.

Proposition 1. The BLSM parameters for the disturbing potential estimator at satellite level are derived by setting $b_n(r) = s_n(r)$ and solving the system of equations Eq. (2e) with the following elements (Sjöberg, 2003):

$$a_{kr} = a_{rk} = (\sigma_r^2 + dc_r)\delta_{kr} - E_{kr}(\psi_0)\sigma_r^2 - E_{rk}(\psi_0)\sigma_k^2 + \sum_{n=2}^{\infty} E_{nr}(\psi_0)E_{nk}(\psi_0)(\sigma_n^2 + c_n)$$