Journal of Geodynamics 49 (2010) 205–209

Contents lists available at ScienceDirect

Journal of Geodynamics

journal homepage: http://www.elsevier.com/locate/jog

The influence of seasonal signals on the estimation of the tectonic motion in short continuous GPS time-series

M.S. Bos[a,*], L. Bastos[b], R.M.S. Fernandes[c,d]

[a] CIIMAR, University of Porto, Rua dos Bragas, 289, 4050-123 Porto, Portugal
[b] Astronomical Observatory, Faculty of Sciences, University of Porto, Monte da Virgem, 4430-146 V.N. Gaia, Portugal
[c] University of Beira Interior, CGUL, IDL, Covilhã, Portugal
[d] Delft Earth-Oriented Space Research (DEOS), DUT, Kluyverweg 1, 2629HS Delft, The Netherlands

ARTICLE INFO

Article history:
Received 24 December 2008
Received in revised form 24 July 2009
Accepted 6 October 2009

Keywords:
GPS
Time-series
Noise

ABSTRACT

Most GPS time-series exhibit a seasonal signal that can have an amplitude of a few millimetres. This seasonal signal can be removed by fitting an extra sinusoidal signal with a period of one year to the GPS data during the estimation of the linear trend.

However, Blewitt and Lavallée (2002) showed that including an annual signal in the estimation process still can give a larger linear trend error than the trend error estimated from data from which the annual signal has been removed by other means. They assumed that the GPS data only contained white noise and we extend their result to the case of power-law plus white noise which is known to exist in most GPS observations. For the GPS stations CASC, LAGO, PDEL and TETN the difference in trend error between having or not having an annual signal in the data is around ten times larger when a power-law plus white noise model is used instead of a pure white noise model. Next, our methodology can be used to estimate for any station how much the accuracy of the linear trend will improve when one tries to subtract the annual signal from the GPS time-series by using a physical model.

Finally, we demonstrate that for short time-series the trend error is more influenced by the fact that the noise properties also need to be estimated from the data. This causes on average an underestimation of the trend error.

1. Introduction

The time-series analysis of daily GPS positions to estimate the secular motion of discrete points due to plate tectonics, has become a routine operation. Examples are the routinely processing of the European Permanent Network of EUREF (Bruyninx, 2004), and the daily processing carried out at the Scripps Orbit and Permanent Array Center (Prawirodirdjo and Bock, 2004). At both initiatives hundreds of permanent GPS stations are analysed in a semi-automatic manner.

Mostly a linear plate motion, also called a linear trend, is assumed. In addition, the fact that these daily GPS observations are correlated in time is nowadays usually taken into account. This results in a larger uncertainty of the estimated linear trend compared to the formal error of uncorrelated observations (Johnson and Agnew, 1995).

It has been observed that most GPS position time-series exhibit an annual signal with an amplitude of a few millimetres (e.g., Dong et al., 2002). For 27 GPS stations located over Iberia, taken from the WEGENER's GEOdynamic Data and Analysis Center (GEODAC), we found on average 1 mm amplitude in North and East component and 2 mm in the vertical component.

Although the causes are not yet completely understood, the most likely explanations are a combination of atmospheric loading (e.g., van Dam et al., 1997), hydrological loading (e.g., van Dam et al., 2001), and thermal expansion of the GPS stations (e.g., Romagnoli et al., 2003).

Blewitt and Lavallée (2002) demonstrated that an annual signal within the data deteriorates the accuracy of the estimated linear trend in time-series with an observation time span of a few years, even when this annual signal is taken into account during the estimation process.

They also showed that the presence of an annual signal has no influence on the accuracy of the linear trend when the observation period is around 1.5, 2.5, 3.5 and 4.5 years. At these specific observation periods there is no correlation between the annual signal and the linear trend. For longer observation spans the correlation between the annual signal and the linear trend can be neglected.

However, their results are based on the assumption of having white noise in the GPS observations while we already mentioned

* Corresponding author. Fax: +351 223 390 608.
E-mail address: mbos@ciimar.up.pt (M.S. Bos).

0264-3707/$ – see front matter © 2009 Elsevier Ltd. All rights reserved.
doi:10.1016/j.jog.2009.10.005

that it is nowadays accepted that GPS noise is correlated in time. In this research we investigate how temporal correlated noise alters the conclusions of Blewitt and Lavallée (2002). In addition, since we also have to estimate the parameters of the noise that is present within the data, we investigate how our imperfect knowledge of their properties influences the estimated trend error.

2. Theory

Let us assume that we have a data set of N daily GPS positions which we designate by the vector $\mathbf{x}$. These could be positions in East, North or Up direction. Since each direction is analysed separately, we only describe the procedure in one component. To these data we fit a linear trend that represents the tectonic motion and an arbitrary offset using Weighted Least-Squares. The design matrix is simply:

$$\mathbf{H} = \begin{pmatrix} 1 & t_0 \\ 1 & t_1 \\ \vdots & \vdots \\ 1 & t_{N-1} \end{pmatrix} \quad \text{with} \quad t_0 = 0, \quad t_1 = 1, \quad \ldots \tag{1}$$

To include an annual signal in the design matrix, we need to modify it as follows:

$$\mathbf{H} = \begin{pmatrix} 1 & t_0 & \cos(\omega_a t_0) & \sin(\omega_a t_0) \\ 1 & t_1 & \cos(\omega_a t_1) & \sin(\omega_a t_1) \\ \vdots & & & \vdots \\ 1 & t_N & \cos(\omega_a t_N) & \sin(\omega_a t_N) \end{pmatrix} \tag{2}$$

where ω_a is the angular velocity of the annual signal.

If we also assume that we know the covariance matrix of the noise that is present within the signal, designated by $\mathbf{C}$, then the covariance matrix of the estimated parameters $\hat{\theta}$ is (Kay, 1993):

$$\text{Cov}(\hat{\theta}) = (\mathbf{H}^T \mathbf{C}^{-1} \mathbf{H})^{-1} \tag{3}$$

Eq. (3) does not account for modelling errors. If for example the data would contain a quadratic signal (the tectonic plate is accelerating) while we do not account for it in our design matrix, then the fit between the model and observations would be worse that it could be. This normally results in larger values in the covariance matrix $\mathbf{C}$, which is estimated from the misfit between model and observations. As a result, the uncertainty of the estimated parameters is larger than it could be. The better we model the signal, the better is the accuracy of the estimated parameters.

On the other hand, if we try to fit a too complex model to the data with many parameters there is the risk that the accuracy of the estimated parameters decreases. This happens when there is a large correlation between the different parameters which complicates their separation.

The separation of different signals within the data is also less accurate when the noise is correlated in time. If the noise is not correlated in time then it is called white noise, otherwise it is called coloured noise. When the spectrum of the noise is of the form $1/f^\alpha$, where f is the frequency, one speaks about power-law noise. The parameter α is called the spectral index. Caporali (2003) and Williams et al. (2004), among others, have shown that the noise in GPS data can be well described as the sum of white and power-law noise.

As a result, the covariance matrix of the noise can be written as (Williams, 2003b):

$$\mathbf{C} = \sigma_w^2 \mathbf{I} + \sigma_{pl}^2 \mathbf{E}(\alpha) \tag{4}$$

where σ_w^2 and σ_{pl}^2 are the variances of the white and power-law noise. $\mathbf{I}$ is the unit matrix and $\mathbf{E}$ represents the covariances due to the power-law noise and depends on the spectral index α.

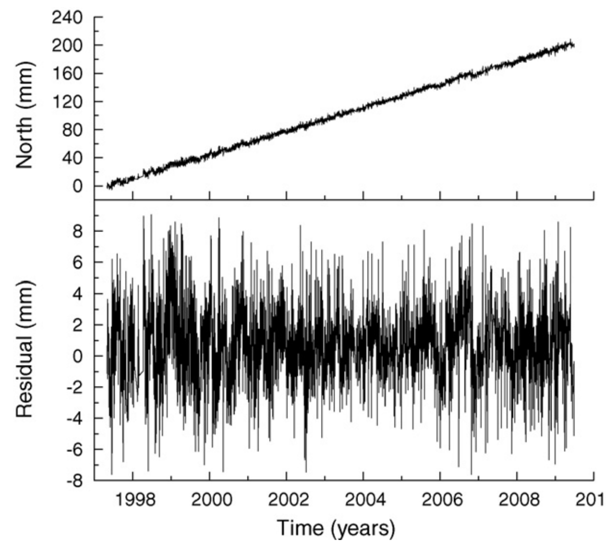

Fig. 1. The upper panel shows the North component of the GPS position time-series at CASC. The lower panel shows the same data set after subtraction of a linear trend and a yearly signal.

3. Observations

To illustrate the effects related to analysing short time-series we used GPS data from four stations of which two are in Portuguese Mainland: CASC and LAGO. The other stations are PDEL, Ponta Delgada in the Azores, and TETN which is situated in the North of Morocco. The first three time-series have an observation span of longer than nine years while TETN only has 2.4 years of continuous GPS observations. Only the last station can be considered to be a short time-series but we will use the longer time-series to validate our results.

Data have been processed using JPL's GIPSY software package (Webb and Zumberge, 1995), following the precise point positioning strategy which resulted in time-series of daily coordinate solutions. These time-series were afterwards converted to the ITRF2005 reference frame (Altamimi et al., 2007) and filtered to remove outliers. Offsets were estimated at epochs when the GPS receiver or antenna were replaced.

For CASC, the time-series of the North component is presented in Fig. 1. The power spectral density of the differences between observations and the fitted model, is given in Fig. 2. In this last

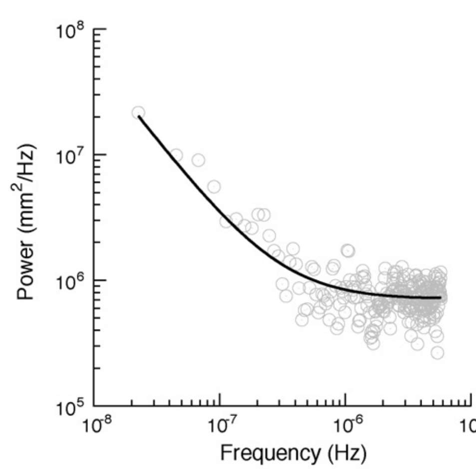

Fig. 2. Power spectral density of the GPS residuals at CASC, North component. The circles denote the spectral density computed from the observations and the solid line is the fitted power-law plus white noise model.