Journal of Geodynamics 48 (2009) 331–339

Contents lists available at ScienceDirect

Journal of Geodynamics

journal homepage: http://www.elsevier.com/locate/jog

Estimation of the Free Core Nutation parameters from SG data: Sensitivity study and comparative analysis using linearized least-squares and Bayesian methods

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A R T I C L E I N F O

Keywords:
Free Core Nutation
Superconducting Gravimeter
Bayesian analysis

A B S T R A C T

The Free Core Nutation (FCN) is investigated with the help of its resonance effect on the tidal amplitudes in Superconducting Gravimeter (SG) records of the GGP network. The FCN resonance parameters are combined in a resonance equation involving the Earth's interior parameters. The sensitivity of the FCN parameters to the diurnal tidal waves demonstrates that the quality factor of the FCN is strongly dependent on the accuracy of the imaginary part estimates of the gravimetric factors close to the resonance. The weak amplitude of Ψ_1 tidal wave on the Earth, which is the closest in frequency to the FCN, in addition to errors in ocean loading correction, explains the poor determination of the quality factor Q from surface gravimetric data. The inversion of tidal gravimetric factors leads to estimates of the period, Q and resonance strength of the FCN. We show that, by inverting log(Q) instead of Q, the results using the least-squares method optimized using the Levenberg–Marquardt algorithm are in agreement with the Bayesian probabilistic results and agree with the results obtained from VLBI nutation data. Finally, a combined inversion of 7 GGP European SG data is performed giving $T = 428 \pm 3$ days and $7762 < Q < 31{,}989$ (90% C.I.). An experimental estimate of the internal pressure Love number is also proposed.

1. Introduction

Because of the fluidity of the core, the Earth has a rotational mode, called the Free Core Nutation (FCN) with a period almost diurnal in Earth-fixed coordinates. The FCN parameters (period, damping) strongly depend on the coupling mechanism at the core–mantle boundary (flattening, topography, electro-magnetic coupling...). The FCN can be detected by its effect on the Earth's rotation, using the VLBI network analyses, or by studying its effects on the gravity field. As the tidal potential contains some diurnal components, a resonance occurs in the diurnal frequency band. This resonance effect can be observed in time-varying gravity data continuously recorded on the Earth's surface by Superconducting Gravimeters (SGs) of the Global Geodynamics Project (GGP) network (Crossley et al., 1999). The FCN resonance in gravity data is commonly represented by a damped harmonic oscillator model that we invert in order to determine the FCN frequency, quality factor Q and the transfer function of the mantle (or the resonance strength). The usual approach to solve this non-linear inverse problem is to use a linearized least-squares method optimized based on the Levenberg–Marquardt algorithm (Marquardt, 1963 – Numerical Recipes Fortran Chapter 15.5 – see for instance Defraigne et al., 1994, 1995; Sato et al., 2004; Ducarme et al., 2007). However Florsch and Hinderer (2000) have demonstrated the inadequacy of using such a least-squares method, because the statistical distribution of Q is definitely not Gaussian. They have proposed instead the use of a Bayesian approach to invert the FCN parameters, since the Bayesian method better propagates the information to the parameters.

Neuberg et al. (1987) first proposed an inversion of stacked gravity tide measurements in central Europe to retrieve the FCN parameters using the Marquardt optimized linearized least-squares. Then Defraigne et al. (1994) extended the gravity stack to the nutation observations. In those past studies, the obtained Q-value was abnormally small and sometimes even negative. Sato et al. (2004) used $1/Q$ instead of Q as a parameter to be inverted using a modified Marquardt least-squares method since $1/Q$ seems to be Gaussian (Sato et al., 1994). However they obtained a Q-value still smaller than the one retrieved from the VLBI nutation analysis (Table 1). The first application of the Bayesian method was proposed by Florsch and Hinderer (2000), who introduced log(Q) as a parameter instead of Q, in order to preserve the positivity of Q, and

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doi:10.1016/j.jog.2009.09.027

Table 1
A summary of various estimates of period and quality factor of the FCN. In addition to theoretical results relative to an elastic Earth, to a slightly inelastic one and to MHB2000 model of Mathews et al. (2002), we have added experimental results from the IDA (International Digital Accelerometers) network of spring gravimeters and from VLBI (Very Long Baseline Interferometry). The other results are from superconducting gravimeter (SG) datasets: B, Brussel (Belgium); BH, Bad-Homburg (Germany); ST, Strasbourg (France); CA, Cantley (Canada); J, 3 Japanese stations; ES, Esashi (Japan); MA, Matsushiro (Japan); CB, Canberra (Australia); MB, Membach (Belgium).

Author	Data	T	Q
Neuberg et al. (1987)	Stacked gravity (B + BH)	431 ± 6	2800 ± 500
Sasao et al. (1980)	Theory elastic	465	∝
Wahr and Bergen (1986)	Theory anelastic	474	78,000
Herring et al. (1986)	VLBI	435 ± 1	22,000–10^5
Cummins and Wahr (1993)	Stacked gravity IDA	428 ± 12	3300–37,000
Sato et al. (1994)	Stacked gravity J	437 ± 15	3200–∝
Defraigne et al. (1994)	Stacked gravity	424 ± 14	2300–8300
	VLBI	432 ± 4	Q > 15,000
	Stacked gravity + VLBI	433 ± 3	Q > 17,000
Florsch et al. (1994)	Gravity ST	431 ± 1	1700–2500
Merriam (1994)	Gravity CA	430 ± 4	5500–10,000
Hinderer et al. (1995)	Stacked gravity (ST + CA)	429 ± 8	7700–∝
Roosbeek et al. (1999)	VLBI	431–434	–
Florsch and Hinderer (2000)	Gravity ST (Bayes)	428	Q > 20,000
Hinderer et al. (2000)	Gravity + VLBI	431–434	15,000–30,000
Mathews et al. (2002)	MHB2000 model	430.20 ± 0.28	20,000
Sato et al. (2004)	Stacked gravity (ES + MA + CB + MB)	429.7 ± 1.4	9350–10,835
Vondràk and Ron (2006)	VLBI	430.32 ± 0.07	20,600 ± 340
Ducarme et al. (2007)	Mean gravity	429.7 ± 2.4	Not estimated
Lambert and Dehant (2007)	VLBI	430 ± 0.4	17,000 ± 3000
Ducarme et al. (2009)	Mean gravity in Europe	430 ± 2	15,000 ± 8000
Koot et al. (2008)	VLBI (Bayes)	430	13,750 ± 514
This paper	Stacked gravity of 7 European SGs (Bayes)	428 ± 3	7762 < Q < 31,989 (90% C.I.)

obtained a Q-value greater than 20,000. More recently, Ducarme et al. (2009) inverted log(Q) using both a Bayesian and a least-squares approach but applied on averaged gravimetric factors from European sites. They obtained a value for Q consistent with the VLBI result. The other studies based on the least-squares method are summarized in Table 1. Note that Koot et al. (2008) performed an estimation of the FCN resonance parameters from VLBI nutation series using a Bayesian statistical approach in the time domain.

Here we propose a comparison of the results given by the linearized least-squares method optimized by the Levenberg–Marquardt algorithm with the Bayesian inversion applied on SG gravity records. We show that the Q-value obtained from SG data is now in agreement with the value inverted from VLBI nutation series whatever the method used, least-squares or Bayesian inversion. Besides, we demonstrate that the poor constraint on the Q-value obtained by Florsch and Hinderer (2000) was due to the large uncertainty on the phase of the diurnal tidal waves close to the resonance.

In the first part we describe the FCN resonance model. Then, we review the theory of the Bayesian method and the Levenberg–Marquardt optimization algorithm applied to linearized least-squares. A qualitative study is then performed to check the sensitivity of the gravity factors to the FCN parameters. Finally, we invert the FCN resonance parameters using a combination of 7 European SG time-series.

2. The FCN resonance model

The basic equation used to describe the resonance of the FCN in the tidal gravity is usually written as (Hinderer et al., 1991a):

$$\tilde{\delta}_j = \tilde{\delta}_{ref} + \frac{\tilde{a}}{\sigma_j - \tilde{\sigma}_{nd}}, \quad (1)$$

where $\tilde{\delta}_j$ corresponds to the complex gravimetric factor observed for every tidal wave of frequency σ_j, $\tilde{\sigma}_{nd} = \sigma^R_{nd} + i\sigma^I_{nd}$ is the complex eigenfrequency of the FCN, $\tilde{a} = a^R + ia^I$ refers to the resonance strength corresponding to the response of the whole Earth to the FCN. The quantity $\tilde{\delta}_{ref}$ is the value of the gravimetric factor without any resonance process (classical tidal gravimetric factor); it is also the asymptotic value of $\tilde{\delta}_j$ for frequencies far away from the resonance frequency. The eigenperiod T of the FCN expressed in sidereal days in the rotating frame is related to σ^R_{nd} by

$$T = \frac{2\pi}{\sigma^R_{nd}},$$

where σ^R_{nd} is expressed in radian per sidereal day. In the inertial reference frame, the period can be written:

$$T' = \frac{1}{\sigma^R_{nd}C - 1}$$

where $C = 86{,}164/15 \times 86{,}400$ and σ^R_{nd} is given in degrees/solar hour. The quality factor Q, expressing the damping due to all the physical processes involved in the resonance, is defined as $Q = \sigma^R_{nd}/2\sigma^I_{nd}$. The quantities σ^R_{nd} and σ^I_{nd} are positive by definition, therefore they should follow a log-normal distribution law (Tarantola, 2005; Florsch and Hinderer, 2000) to avoid possible negative values. It is therefore recommended to include the *a priori* positivity of Q in the model by changing the variable $Q = 10^x$ and inverting for x, instead of Q.

Florsch and Hinderer (2000) performed the inversion by treating $\tilde{\delta}_{ref}$ as an unknown and showed that a correlation exists between the real parts δ^R_{ref} and σ^R_{nd}, and between a^R and δ^R_{ref}, but the correlation between a^R and T is much stronger. As $\tilde{\delta}_{ref}$ has a weak influence on the values of T and Q, we do not include this parameter in the inversion process. In previous studies (e.g. Defraigne et al., 1994, 1995; Ducarme et al., 2007), the observed value for the tidal wave O_1 was used as the reference gravimetric factor. In our case we will use the mean value of the theoretical inelastic amplitude factors of the O_1 and OO_1 waves computed for the Wahr–Dehant model (Wahr, 1974; Dehant, 1987). By doing so, we suppose that the scale factors of the SG used here are accurate enough, which is usually true (better than 0.3% accuracy, e.g. Amalvict et al., 2001; Sato et al., 2004). We could also have normalized by the observed O1 amplitude as done by Sato et al. (2004) but we suppose that the scaling error is negligible with respect to the ocean loading uncertainty.