Computers and Mathematics with Applications 61 (2011) 1676–1681

Contents lists available at ScienceDirect

Computers and Mathematics with Applications

journal homepage: www.elsevier.com/locate/camwa

Series solution of a nonlinear ODE arising in magnetohydrodynamic by HPM-Padé technique

Behrouz Raftari [a], Ahmet Yildirim [b,*]

[a] Department of Mathematics, Islamic Azad University, Kermanshah branch, P.C. 6718997551, Kermanshah, Iran

[b] Department of Mathematics, Ege University, 35100, Bornova, Izmir, Turkey

ARTICLE INFO

Article history:
Received 5 October 2010
Accepted 27 January 2011

Keywords:
Homotopy perturbation method
Padé approximation
MHD viscous flow
Nonlinear ODE

ABSTRACT

In this study, we investigate the magnetohydrodynamic (MHD) viscous flow due to a shrinking sheet by employing the homotopy perturbation method (HPM) and Padé approximation. The series solution of the governing nonlinear problem is developed. Generally, the truncated series solution is adequate only in a small region when the exact solution is not reached. We overcame this limitation by using the Padé techniques, which have the advantage in turning the polynomial approximation into a rational function, are applied to the series solution to improve the accuracy and enlarge the convergence domain. Comparison of the present solutions is made with the results obtained by other applied methods and excellent agreement is noted.

1. Introduction

Analytical methods have made a comeback in research methodology after taking a backseat to the numerical techniques for the latter half of the preceding century. The advantages of analytical methods are manifold, the main being that they give a much better insight than the numbers crunched by a computer using a purely numerical algorithm. Two of the analytical methods of recent vintage, namely, the homotopy perturbation method (HPM) and the homotopy analysis method (HAM), have attracted special attention from researchers as they are both flexible in applying and give sufficiently accurate results with modest effort. Both the methods are based upon introduction of a homotopy parameter p which takes the values from zero to one. When $p = 0$, the problem under study takes a sufficiently simple form which admits a closed form analytical solution. As p is increased and finally takes the value one, the solution to the original problem is recovered, the nice feature is that it is being done entirely analytically.

The HPM was introduced by He [1] in his endeavor to solve nonlinear problems using the perturbation technique. Rather than depending on one of the physical parameters being small for being chosen as a perturbation parameter, He proposed to introduce p, which in the spirit of homotopy is increased from 0 to 1. There is again a whole gamut of variations which makes the method quite attractive and amenable for applying to numerous kinds of applications. He [2–10] has published extensively demonstrating the viability of his method. The latest intricacies and variations of the HPM with its application to the nonlinear problems have been covered in a monograph by He [11]. Recently, the homotopy perturbation method has been used to solve a wide range of problems [12–20].

The boundary layer viscous flow induced by stretching the surface moving with a certain velocity in an otherwise quiescent fluid medium often occurs in several engineering processes. Such flows have promising applications in industries, for example in the extrusion of a polymer sheet from a die or in the drawing of plastic films. During the manufacture of these

* Corresponding author.
E-mail addresses: ahmet.yildirim@ege.edu.tr, ahmetyildirim80@gmail.com (A. Yildirim).

0898-1221/$ – see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.camwa.2011.01.037

sheets, the melt issues from a slit and is subsequently stretched to achieve the desired thickness. The mechanical properties of the final product strictly depend on the stretching and cooling rates in the process.

Since the pioneering work of Sakiadis [22,23] various aspects of boundary layer flow due to a stretching sheet have been investigated by several workers in the field. Specifically Cranes problem [24] for flow of an incompressible viscous fluid past a stretching sheet has become a classic in the literature. It admits an exact analytical solution. Besides it has produced a galore of associated problems, each incorporating a new effect and still giving an exact solution. The uniqueness of the exact analytical solution presented in [24] is discussed by McLeod and Rajagopal [25]. Gupta and Gupta [26] examined the stretching flow subject to suction or injection. The flow inside a stretching channel or tube has been analyzed by Brady and Acrivos [27] and the flow outside the stretching tube by Wang [28]. In another paper, Wang [29] extended the flow analysis to the three-dimensional axisymmetric stretching surface. The unsteady flow induced by stretching film have been also discussed by Wang [30] and Usha and Sridharan [31].

All the above mentioned investigations deals with the stretching flow problems. But the literature on the shrinking flow problem in very scarce. To the best of our knowledge, only two such attempts [30,32] are yet available in the literature. In [30], Wang presented unsteady shrinking film solution and in [32], Miklavcic and Wang proved the existence and uniqueness for steady viscous hydrodynamic flow due to a shrinking sheet for a specific value of the suction parameter.

In this study, the solution of the nonlinear problem is obtained by combining homotopy perturbation method and Padé approximation. The HPM-Padé technique has been successfully implemented to solve them by converting the series solutions into the diagonal Padé techniques. Numerical illustrations show that it is a promising tool for solving nonlinear problems.

2. Problem statement

In Cartesian coordinates the continuity and momentum equations for MHD viscous flow are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)-\frac{\sigma B_0^2}{\rho}u, \tag{2}$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\nu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)-\frac{\sigma B_0^2}{\rho}v, \tag{3}$$

$$u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial z}+\nu\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right), \tag{4}$$

where $\nu=\mu/\rho$ is the kinematic viscosity. σ is the electrical conductivity. We have applied the magnetic field B_0 in the z-direction and the induced magnetic field is neglected. The above equations are derived by considering the zero electric field and incorporating the small magnetic Reynold number assumption.

The boundary conditions applicable to the present flow are

$$\begin{aligned}&u=-ax,\qquad v=-a(m-1)y,\qquad w=-W,\quad \text{at } y=0,\\&u\to 0\quad \text{as } y\to\infty\end{aligned} \tag{5}$$

in which $a>0$ is the shrinking constant, W is the suction velocity. $m=1$ when sheet shrinks in x-direction only and $m=2$ when the sheet shrinks axisymmetrically. Introducing the following similarity transformations

$$u=axf'(\eta),\qquad v=a(m-1)yf'(\eta),\qquad w=-\sqrt{a\nu}mf(\eta),\qquad \eta=\sqrt{\frac{a}{\nu}}z. \tag{6}$$

Eq. (1) is identically satisfied and Eq. (4) can be integrated to give

$$\frac{p}{\rho}=\nu\frac{\partial w}{\partial z}-\frac{w^2}{2}+\text{constant} \tag{7}$$

and Eqs. (1), (3) and (5) reduces to the following boundary value problem [33]

$$f'''-M^2f'-f'^2+mff''=0, \tag{8}$$

$$\begin{aligned}&f=s,\qquad f'=-1,\quad \text{at } \eta=0,\\&f'\to 0,\quad \text{as } \eta\to\infty,\end{aligned} \tag{9}$$

where $s=W/m\sqrt{a\nu}$ and $M^2=\sigma B_0^2/\rho a$.