Entrainment threshold of sand- to granule-sized sediments under waves

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J.P. Le Roux

Departamento de Geología/Andean Geothermal Centre of Excellence, Facultad de Ciencias Físicas y Matemáticas, Universidad de Chile, Plaza Ercilla 803, Santiago, Chile

A B S T R A C T

An improved method is presented to determine the threshold boundary velocity required to entrain sediments under waves, using the non-dimensional group settling velocity of sediments ranging from very fine sand to granules (0.1–3.3 mm), together with a dimensionless boundary velocity. In combination with a more accurate method to calculate the actual boundary velocity under linear as well as non-linear waves, this allows sediment entrainment to be predicted from deep water up to the breaker zone.

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1. Introduction

Numerous studies have been undertaken on the threshold of sediment entrainment under ocean waves (e.g., Bagnold, 1946; Manohar, 1955; Eagleson et al., 1958; Horikawa and Watanabe, 1967; Rance and Warren, 1968; Komar and Miller, 1973, 1975; Madsen and Grant, 1975; Sleath, 1978; Hammond and Collins, 1979; Hallermeier, 1980; Rigler and Collins, 1983; Soulsby and Whitehouse, 1997; Green, 1999; You, 2000; Le Roux, 2001; Paphitis et al., 2001; You and Yin, 2006). Most of these have focused on the near-bed water particle velocity, ideally measured at the top of the boundary layer where the vertical component of orbital water particle motion reduces to zero. However, because the thickness of the boundary layer cannot be predicted with accuracy, most measurements were probably taken either above the boundary layer, where water particle motion was still ellipsoidal, or below its top, where the measured velocity would have been less than the actual boundary velocity. As a consequence, plots of predicted against measured velocities inevitably display a large scatter of data points for all empirical equations. The method presented in this paper, being based on published data, is no exception, but shows an improved correlation between predicted and measured critical boundary velocities and also provides a way to determine the actual boundary velocity under both linear and non-linear waves.

2. Methodology

2.1. Critical boundary velocity

Many of the existing threshold equations incorporate either the orbital diameter d_o (Bagnold, 1946; Komar and Miller, 1973, 1975) or the water particle semi-excursion at the top of the boundary layer (Wang, 2007), but others employed a Shields-type parameter (Rance and Warren, 1968; Soulsby and Whitehouse, 1997). Le Roux (2001) used a dimensionless boundary velocity ($U_{d\delta}$) in combination with the dimensionless settling velocity (U_{dw}) of spheres having the same diameter as the median sediment size. The results were compared with the equations of Bagnold (1946), Manohar (1955), Komar and Miller (1973, 1975), Hammond and Collins (1979), and Soulsby and Whitehouse (1997) using the data sets of Bagnold (1946), Manohar (1955), and Hammond and Collins (1979). Although it showed a significant improvement, this equation was not dimensionally correct, as it employed a second order polynomial trend-line to further improve the original, dimensionally correct equation. The use of a dimensionless sphere settling velocity is also not ideal, because the group settling velocity of differently shaped grains (which would directly control their entrainment behaviour) is significantly different from the settling velocity of individual spheres (Le Roux, 2014).

You and Yin (2006) subsequently published a unified equation to determine the threshold of sediment entrainment and sheet flow under waves, which gives better results than that of Le Roux (2001) for the same data sets. However, their equation is also not dimensionally correct, in that they use the "dimensionless" term $(s - 1)$, where s is in fact the sediment density and 1 the water density. This makes the use of

E-mail address: jroux@ing.uchile.cl.

their equations questionable in the case of entrainment by waves in sea water instead of fresh water.

Recently, Le Roux (2014) published equations to determine the settling velocity of individual, differently shaped clasts with known axial dimensions, as well as the group settling velocity of sieve-sized sediments (subscript v) with unknown axial dimensions. The latter is given by

$$
\begin{aligned}
\log_{10} U_{dwv} = {} & 0.0195(\log_{10} D_{dv})^5 - 0.0075(\log_{10} D_{dv})^4 - 0.1679(\log_{10} D_{dv})^3 \\
& - 0.1936(\log_{10} D_{dv})^2 + 1.9606(\log_{10} D_{dv}) - 1.2582,
\end{aligned} \tag{1}
$$

where U_{dwv} is the dimensionless group settling velocity, D_v and D_{dv} are the grain size and dimensionless (subscript d) grain size as determined by sieve analysis, respectively, given by $D_{dv} = D_v \sqrt[3]{\frac{\rho g \rho_\gamma}{\mu^2}}$, ρ is the fluid density, g is the acceleration due to gravity, ρ_γ is the submerged density (grain density minus fluid density), and μ is the dynamic fluid viscosity. The settling velocity is non-dimensionalized by

$$
U_{dwv} = U_{wv} \cdot \sqrt[3]{\frac{\rho^2}{\mu g \rho_\gamma}}. \tag{2}
$$

The dimensionless settling velocity can be plotted against a dimensionless boundary velocity, established by Le Roux (2001) as

$$
U_{d\delta} = \frac{U_\delta \sqrt{\frac{\rho \mu}{T}}}{g D \rho_\gamma}, \tag{3}
$$

where U_δ is the actual boundary velocity and T is the wave period.

Due to the difficulty in measuring sediment entrainment thresholds under field conditions, especially in the presence of marine currents and other complicating elements, the vast majority of studies have been carried out in the laboratory. Bagnold (1946), for example, studied a bed of particles resting on an oscillating plate that was submerged in a tank of water, observing the frequency and amplitude of the oscillation required to entrain the grains. Unfortunately, most of these studies did not present the actual data, except on graphs that are difficult to read accurately. Therefore, three widely cited case studies with usable data were examined here, namely those of Bagnold (1946), Manohar (1955), and Hammond and Collins (1979).

Plotting U_{dwv} against the measured critical boundary velocities ($U_{d\delta crit}$) for these data sets (Fig. 1), shows that the dimensionless critical boundary velocity can be found by

$$
U_{d\delta crit} = -0.0083 \ln U_{dwv} + 0.0247. \tag{4}
$$

Finally, the critical boundary velocity is given by

$$
U_{\delta crit} = \frac{0.848 U_{d\delta crit} g D_v \rho_\gamma}{\sqrt{\frac{\rho \mu}{T}}}. \tag{5}
$$

The above-mentioned data sets include 209 measurements with grain sizes varying from 0.1 to 8 mm, densities between 1.05 and 7.9 g cm^{-3}, boundary velocities between 4.45 and 47.26 cm s^{-1}, and wave periods between 0.76 and 26.1 s. Fig. 2 compares the boundary velocities predicted by Eq. (5) with the measured velocities. The correlation coefficient R^2 is 0.8044, with a 1:1 relationship between the trend-line of the observed and predicted velocities.

The mean percentage error (*MPE*), given by

$$
MPE = \frac{100 \left(U_{\delta critm} - U_{\delta critp} \right)}{U_{\delta critm}}, \tag{6}
$$

where $U_{d\delta critm}$ and $U_{d\delta critp}$ are the measured and predicted critical boundary velocities, respectively, is 2.95%, with a maximum positive error of 39.48% and maximum negative error of -123.83%. The latter value is that of an obviously anomalous measurement, for which the You and Yin (2006) unified equation also yields a very large error of -222.83%. Without this data point the maximum negative error would be -38.9% for Eq. (5) and -70.12% for You and Yin (2006). The *MPE* for the latter authors is 6.76% and their ratio between the measured and predicted critical boundary velocities is 0.8736, meaning that their equation generally underestimates the critical boundary velocity. For the original equation of Le Roux (2001), the *MPE* is 5.67%, with a maximum positive error of 27.99% and a maximum negative error of -159.77% (-123.21% if the anomalous value is excluded). Eq. (5) thus yields the lowest *MPE* and lowest maximum absolute error of 39.48%, compared to maximum absolute errors of 70.12% and 123.21% for You and Yin (2006); and Le Roux (2001), respectively, again excluding the anomalous value.

2.2. Actual boundary velocity

Although Eq. (5) provides a way to obtain the critical wave boundary velocity, the actual boundary velocity under different wave climates is an entirely different matter, especially under field conditions. To know whether sediments will be entrained in any particular water depth for a specific set of wave conditions, it is necessary to be able to predict the real boundary velocity at that specific depth. Only if the latter exceeds the critical boundary velocity for the specific sediment size and density, will entrainment take place.
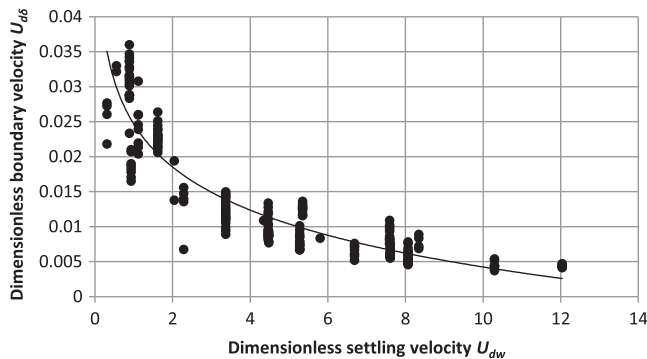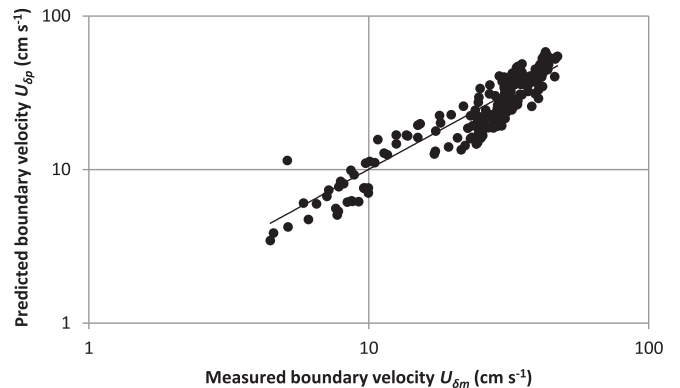
Fig. 1. Plot of dimensionless settling velocity (Eqs. (1) and (2)) against dimensionless critical boundary velocity (Eq. (3)).
Data from Bagnold (1946), Manohar (1955), and Hammond and Collins (1979).

Fig. 2. Plot of predicted (Eq. (5)) against measured critical boundary velocity.
Data from Bagnold (1946), Manohar (1955), and Hammond and Collins (1979).