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Approximations of fuzzy numbers by trapezoidal fuzzy numbers preserving the ambiguity and value

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ABSTRACT

Value and ambiguity are two parameters which were introduced to represent fuzzy numbers. In this paper, we find the nearest trapezoidal approximation and the nearest symmetric trapezoidal approximation to a given fuzzy number, with respect to the average Euclidean distance, preserving the value and ambiguity. To avoid the laborious calculus associated with the Karush–Kuhn–Tucker theorem, the working tool in some recent papers, a less sophisticated method is proposed. Algorithms for computing the approximations, many examples, proofs of continuity and two applications to ranking of fuzzy numbers and estimations of the defect of additivity for approximations are given.

1. Introduction

Uncertainty and incomplete information in decision making, linguistic controllers, expert systems, data mining, pattern recognition, etc., are often represented by fuzzy numbers. In recent years, several researchers have focused on the computing of different approximations of fuzzy numbers and new approaches for ranking of fuzzy numbers (see, e.g., [1–17]).

To capture the relevant information, to simplify the task of representing and handling fuzzy numbers, the value and the ambiguity of a fuzzy number were introduced in [18]. In the same paper, the authors discussed how to approximate a given fuzzy number by a suitable trapezoidal one preserving the value and ambiguity. Because it is not possible to uniquely determine a trapezoidal fuzzy number, which is characterized by four numbers, from two conditions, some additional conditions must be introduced. In the present paper, we completely solve the problems of finding the nearest trapezoidal approximation and the nearest symmetric trapezoidal approximation of a fuzzy number with respect to the average Euclidean distance, such that the value and ambiguity are preserved.

The paper is organized as follows. In Section 2, we recall basic definitions and results. The nearest trapezoidal approximation of a fuzzy number preserving the value and ambiguity is determined in Section 3. The method is less sophisticated than previous methods; it avoids the laborious calculus associated with the Karush–Kuhn–Tucker theorem and, in addition, it allows us to prove the continuity of the trapezoidal approximation. The symmetric case is tackled in

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Section 4. Taking into account the solving in the general case, the results are easily obtained. The expected value, width, left-hand ambiguity and right-hand ambiguity are parameters used in Section 5 to express the main results of the paper in a more compact form and to give some algorithms for calculating the trapezoidal approximations. Sections 6 and 7 are dedicated to examples and properties. We insist on the continuity of the trapezoidal approximation operators and we ignore the elementary proofs of other properties. Value and ambiguity are used together to rank fuzzy numbers [18]; therefore we can compare the trapezoidal approximations preserving the value and ambiguity instead, to compare fuzzy numbers (Section 8). The defects of additivity of trapezoidal approximation operators determined in Sections 3 and 4 are estimated in Section 8. The paper is completed by some conclusions and open problems.

2. Preliminaries

We consider the following well-known description of a fuzzy number A:

$$A(x) = \begin{cases} 0, & \text{if } x \le a_1, \\ l_A(x), & \text{if } a_1 \le x \le a_2, \\ 1 & \text{if } a_2 \le x \le a_3, \\ r_A(x), & \text{if } a_3 \le x \le a_4, \\ 0, & \text{if } a_4 \le x, \end{cases} \tag{1}$$

where $a_1, a_2, a_3, a_4 \in \mathbb{R}$, $l_A : [a_1, a_2] \longrightarrow [0, 1]$ is a non-decreasing upper semicontinuous function, $l_A(a_1) = 0$, $l_A(a_2) = 1$, called the left side of the fuzzy number and $r_A : [a_3, a_4] \longrightarrow [0, 1]$ is a non-increasing upper semicontinuous function, $r_A(a_3) = 1$, $r_A(a_4) = 0$, called the right side of the fuzzy number. The α-cut, $\alpha \in (0, 1]$, of a fuzzy number A is a crisp set defined as

$$A_\alpha = \{x \in \mathbb{R} : A(x) \ge \alpha\}.$$

The support or 0-cut A_0 of a fuzzy number is defined as

$$A_0 = \overline{\{x \in \mathbb{R} : A(x) > 0\}}.$$

Every α-cut, $\alpha \in [0, 1]$, of a fuzzy number A is a closed interval

$$A_\alpha = [A_L(\alpha), A_U(\alpha)],$$

where

$$A_L(\alpha) = \inf\{x \in \mathbb{R} : A(x) \ge \alpha\},$$
$$A_U(\alpha) = \sup\{x \in \mathbb{R} : A(x) \ge \alpha\},$$

for any $\alpha \in (0, 1]$. If the sides of the fuzzy number A are strictly monotone, then one can see easily that A_L and A_U are inverse functions of l_A and r_A, respectively. We denote by $F(\mathbb{R})$ the set of all fuzzy numbers.

Some important parameters of a fuzzy number A, $A_\alpha = [A_L(\alpha), A_U(\alpha)]$, $\alpha \in [0, 1]$, are the ambiguity $\mathrm{Amb}(A)$ and the value $\mathrm{Val}(A)$. They are given by (see [18])

$$\mathrm{Amb}(A) = \int_0^1 \alpha(A_U(\alpha) - A_L(\alpha))\mathrm{d}\alpha, \tag{2}$$

$$\mathrm{Val}(A) = \int_0^1 \alpha(A_U(\alpha) + A_L(\alpha))\mathrm{d}\alpha. \tag{3}$$

A metric on the set of fuzzy numbers, which is an extension of the Euclidean distance, is defined by [19]

$$d^2(A, B) = \int_0^1 (A_L(\alpha) - B_L(\alpha))^2\mathrm{d}\alpha + \int_0^1 (A_U(\alpha) - B_U(\alpha))^2\mathrm{d}\alpha. \tag{4}$$

Fuzzy numbers with simple membership functions are preferred in practice. The most used such fuzzy numbers are so-called trapezoidal fuzzy numbers. A trapezoidal fuzzy number T, $T_\alpha = [T_L(\alpha), T_U(\alpha)]$, $\alpha \in [0, 1]$, is given by

$$T_L(\alpha) = t_1 + (t_2 - t_1)\alpha$$

and

$$T_U(\alpha) = t_4 - (t_4 - t_3)\alpha,$$

where $t_1, t_2, t_3, t_4 \in \mathbb{R}$, $t_1 \le t_2 \le t_3 \le t_4$. When $t_2 = t_3$, we obtain a triangular fuzzy number. When $t_2 - t_1 = t_4 - t_3$, we obtain a symmetric trapezoidal fuzzy number. We denote

$$T = (t_1, t_2, t_3, t_4)$$