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Systematic study of the effects of mass and time scaling techniques applied in numerical rock mechanics simulations

Thomas Heinze [a,*], Gunnar Jansen [b], Boris Galvan [b], Stephen A. Miller [b]

[a] University of Bonn, Meckenheimer Allee 176, 53115 Bonn, Germany
[b] CHYN, University of Neuchatel, Batiment UniMail, Rue Emile-Argand 11, 2000 Neuchatel, Switzerland

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ABSTRACT

Numerical modeling is a well established tool in rock mechanics studies investigating a wide range of problems. Implicit methods for solving linear equations have the advantage of being unconditionally stable, while explicit methods, although limited by the time step, are often used because of their limited memory demand, their scalability in parallel computing, and simple implementation of complex boundary conditions. In numerical modeling of explicit elastoplastic dynamics where the time step is limited by the material density, mass scaling techniques can be used to overcome this limit and significantly reduce computation time. While often used, the effect of mass and time scaling and how it may influence the numerical results is rarely-mentioned in publications, and choosing the right scaling technique is typically performed by trial and error. To our knowledge, no systematic studies have addressed how mass scaling might affect the numerical results. In this paper, we present results from an extensive and systematic study of the influence of mass and time scaling on the behavior of a variety of rock-mechanical models. We employ a finite difference scheme to model uniaxial and biaxial compression experiments using different mass and time scaling factors, and with physical models of increasing complexity up to a cohesion-weakening frictional-strengthening model (CWFS). We also introduce a normalized energy ratio to assist analyzing mass scaling effects. We find the tested models to be less sensitive to time scaling than to mass scaling, so mass scaling has higher potential for decreasing computational costs. However, we also demonstrate that mass scaling may lead to quantitatively wrong results, so care must be taken in interpreting stress values when mass scaling is used in complicated rock mechanics simulations. Mass scaling significantly influences the stress–strain response of numerical rocks because mass scaling acts as an artificial hardening agent on rock deformation.

1. Introduction

The field of numerical rock mechanics is well-established, with many different numerical methods applied to a wide range of problems (e.g. Rozhko, 2007; Popov and Sobolev, 2008; Galvan and Miller, 2013; Choi et al., 2013; Azevedo and Lemos, 2013; Tang et al., 2013). Continuum mechanical approaches are typically applied if problems relate to the dynamical response of a rock mass. In these cases, when the interest is on the time evolution of the rock deformation, and not just its final state, there are mainly two approaches: either solving the problem of elastoplastic deformation in a full non-equilibrium dynamical way, explicitly describing the dynamic part of the equations, or alternatively describing the process as a series of small equilibrium steps, implicitly accounting for the dynamics. The latter approach is also commonly known as quasi-static deformation, where the load is incrementally applied and the corresponding equilibrium state for every loading step is successively computed until the full load is reached. If the full dynamical solution is required, the problem is discretized in time by an explicit or implicit scheme. While most implicit time discretization schemes are unconditionally stable with respect to time and space discretization, these schemes are burdened by high computational costs and memory demands. This is particularly the case in 3D where implicit solutions require solving a prohibitively large system of linear equations. Explicit time descritization schemes do not require a LSE (linear system of equations) solver, does not impose any additional difficulties on parallelization, and its memory demand grows linearly with size of the problem. An additional advantage is that complex kinematic boundary conditions, such as sliding, can be easily implemented. The primary disadvantage of explicit Eulerian schemes is a limit on the maximum time step by the Courant–Friedrichs–Levy (CFL) stability criteria, and mass scaling methods have been developed to overcome this limitation (e.g. Zhang and Zhang, 2009; Schmidt and Hattel, 2005; Wang et al., 2004).

Scaling techniques are not necessary, and even obstructive, if the dominant time scale of the problem under consideration is in the range of the rock deformation time scale as, for example, in the case of

* Corresponding author. Tel.: +49 228 73 60630.
E-mail addresses: heinze@geo.uni-bonn.de (T. Heinze), gunnar.jansen@unine.ch (G. Jansen), boris.galvan@unine.ch (B. Galvan), stephen.miller@unine.ch (S.A. Miller).

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seismic wave propagation, where information would be lost. The dominant time scale of coupled fluid–rock interactions, on the other hand, is orders of magnitude longer than rock deformation alone, making computation times prohibitively long if no scaling techniques are used (Cundall, 1982).

The maximum time step for an explicit solution scheme is limited by the P-wave velocity V_p and the CFL-criteria. For a domain with grid spacing dx, dy and dz it is (Virieux, 1986):

$$dt \le \left(V_p \sqrt{\frac{1}{dx^2} + \frac{1}{dy^2}} \right)^{-1} \text{(in 2D)} \tag{1}$$

and

$$dt \le \left(V_p \sqrt{\frac{1}{dx^2} + \frac{1}{dy^2} + \frac{1}{dz^2}} \right)^{-1} \text{(in 3D)} \tag{2}$$

where

$$V_p = \sqrt{\frac{\lambda + 2G}{\rho_0}} \tag{3}$$

where λ is the first Lame constant, G is shear modulus and ρ_0 is material density. By replacing the material density ρ_0 with a computational density $\rho_c = \alpha * \rho_0$ to introduce a scaling factor $\alpha > 1$, the maximum allowed time step is increased, and is termed mass scaling (e.g. Jung, 1998; Papeleux and Ponthot, 2002; Kang et al., 2004; Carlberger and Stigh, 2007; Tu and Andrade, 2008; Kazanci and Bathe, 2012). The real material density is not relevant for quasi-static processes as long as inertial forces are low compared to other forces acting, and computational density does not affect body forces. Computational densities can be chosen to be much larger than typical crustal densities ranging from $2000^{kg}/_{m^3}$ and $3000^{kg}/_{m^3}$, (e.g. Tang et al., 2013; Henrard et al., 2007; Mohebbi and Albarzadeh, 2010; Schaare et al., 2008).

Another approach to reduce computation times of numerical rock deformation processes is simply to increase the loading rate, an approach known as time scaling (e.g. Wang et al., 2004; Lorentzen et al., 1998).

While both methods are widely used in geotechnics and geomechanics simulations since the pioneering work of Cundall (1982), no systematic study on these scaling techniques, and their effects on the simulation results, have been performed in a geotechnical context.

Several studies have looked at mass scaling in an engineering context, such as Chung et al. (1998), who studied the influence of mass scaling on the dynamics of a Finite Element (FE) simulation of metal deformation. A criterion was developed in the work of Chung to detect critical use of mass scaling, and a similar criterion was confirmed by Kim et al. (2002) where they studied hydro-forming processes of metal and compared implicit and explicit schemes. Cocchetti et al. (2013) and Olovsson et al. (2005) presented more complex adaptive mass scaling methods based on the filtering of high frequency contributions in the velocity, which are damped without influencing macroscopic deformation. In adaptive mass scaling, the computational higher density is adjusted based on the outcome of the simulation. Cundall (1982) suggests the maximum out-of balance force at each time step as a limit value to increase or decrease the computational density and therefore increasing or decreasing the time step.

Scaling techniques are used one way or another in most modern commercial software packages such as ANSYS or LS-DYNA. The Livermore Software Technology Corporation, which develops LS-DYNA, holds US-Patents about mass scaling techniques (Corporation LST, 2012).

In this paper, we present a systematic study of a rock-mechanical model using and combining different scaling techniques to determine how scaling affects the numerical solution. We investigate the effect of mass and time scaling together with numerical resolution, physical length scale, boundary conditions, internal forces and complex plasticity models that include hardening and softening. We also investigate the interaction between scaling techniques and the elastic damper, which is used to eliminate elastic wave propagation in quasi-static deformation problems.

2. Theoretical and numerical model

Elastodynamics equations in their velocity–stress form describe the elastic response of a rock skeleton in two dimensions

$$\frac{\partial V_x}{\partial t} = \frac{1}{\rho_c}\left(\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} \right) \tag{4}$$

$$\frac{\partial V_y}{\partial t} = \frac{1}{\rho_c}\left(\frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{xy}}{\partial x} \right) - g \tag{5}$$

$$\frac{\partial \sigma_{xx}}{\partial t} = (\lambda + 2\mu)\frac{\partial V_x}{\partial x} + \lambda \frac{\partial V_y}{\partial y} \tag{6}$$

$$\frac{\partial \sigma_{yy}}{\partial t} = \lambda \frac{\partial V_x}{\partial x} + (\lambda + 2\mu)\frac{\partial V_y}{\partial y} \tag{7}$$

$$\frac{\partial \sigma_{xy}}{\partial t} = \mu\left(\frac{\partial V_x}{\partial y} + \frac{\partial V_y}{\partial x} \right) \tag{8}$$

where μ and λ are Lamé constants, ρ_c is the computational density, V_x and V_y are x- and y-components of the velocity vector, σ_{xx}, σ_{yy}, σ_{xy} are components of the stress tensor, and g is gravitational acceleration.

Plastic deformation of rocks is modeled using a Mohr–Coulomb criterion

$$F = \tau - \left(\sigma_m + \frac{C^*}{\tan(\varphi^0)} \right) \cdot \sin(\varphi^*) \tag{9}$$

where F is the yield function, C^* is a mobilized cohesion, φ^* is a mobilized internal frictional angle, φ^0 is the maximal internal frictional angle, τ is the stress deviator, and σ_m is the mean stress (Vermeer and de Borst, 1984).

Cohesion and internal friction angles can be mobilized in terms of a cohesion weakening and frictional strengthening model with a dependence on the effective plastic strain $\bar{\varepsilon}_p$ (Hajiabdolmajid et al., 2002). Mobilized values for the friction angle, cohesion and dilatancy angle are calculated following previous work (Vermeer and de Borst, 1984).

Plastic strain rates are given by

$$\dot{\epsilon}_{ij}^p = 0 \text{ for } F < 0 \text{ or } F = 0 \text{ and } \dot{F} < 0 \tag{10}$$

$$\dot{\epsilon}_{ij}^p = \lambda^p \frac{\partial q}{\partial \sigma_{ij}} \text{ for } F = 0 \text{ and } \dot{F} = 0. \tag{11}$$

where λ^p is the plastic multiplier, $\dot{F}$ is time derivative of F and q is the flow rule.

Effective plastic strain $\bar{\epsilon}^p$ then follows as

$$\bar{\epsilon}^p = \sqrt{\frac{2}{3} \dot{\epsilon^p}^T \cdot M \cdot \dot{\epsilon^p}} \tag{12}$$

where $\dot{\epsilon^p}$ is the plastic strain rate written as a 3-component vector. The first and second components are the normal components, and the third component is the shear direction. M is a diagonal weighting matrix