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Adaptive anti-synchronization of chaotic systems with fully unknown parameters

M. Mossa Al-sawalha [a,*], M.S.M. Noorani [b], M.M. Al-dlalah [c]

[a] *Faculty of Science, Mathematics Department, University of Hail, Saudi Arabia*

[b] *Center for Modelling & Data Analysis, School of Mathematical Sciences, University Kebangsaan Malaysia, 43600 UKM Bangi, Selangor, Malaysia*

[c] *Faculty of Education, Psychology Department, University of Hail, Saudi Arabia*

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ABSTRACT

This paper centers on the chaos anti-synchronization between two identical or different chaotic systems using adaptive control. The sufficient conditions for achieving the anti-synchronization of two chaotic systems are derived based on Lyapunov stability theory. An adaptive control law and a parameter update rule for unknown parameters are introduced such that the Chen system is controlled to be the Lorenz system. Theoretical analysis and numerical simulations are shown to verify the results.

1. Introduction

Synchronization of chaotic systems was first initiated and recorded by Pecora and Carroll in 1990 [1]. About two decades later, their work had progressed like dominoes effect in numerous fields such as chemical reactions, power converters, biological systems, information processing, secure communications, etc. [2]. The excitement is well comprehended in the academic community as its potential implications and applications are bountiful. Another interesting phenomenon discovered was the anti-synchronization (AS), which is noticeable in periodic oscillators. It is a well-known fact that the first observation of synchronization between two oscillators by Huygens in the seventeenth century was, in fact, an AS between two pendulum clocks. Recent re-investigation of Huygens experiment by Blekhman [3] shows that either synchronization or AS can appear depending on the initial conditions of the coupled pendula. Here, AS can also be interpreted as anti-phase synchronization (APS) [4,5]. In other words, there is no difference between AS and APS for oscillators with identical amplitudes [6]. So far, a wide variety of approaches have been proposed for anti-synchronization of chaos or hyperchaos systems, such as generalized active control [7–10], adaptive control [11,12], nonlinear control [13,14], direct linear coupling [15], separation method [16], etc. Most of the existing methods can anti-synchronize two identical or different chaotic systems with known parameters. However, in practical engineering situations, parameters are probably unknown and may change from time to time. Therefore, how to effectively anti-synchronize two chaotic systems with unknown parameters is an important problem for theoretical research and practical application. Among the aforementioned methods, adaptive control is an effective one for achieving the anti-synchronization of chaotic systems with fully unknown parameters [11,12]. On the basis of the Lyapunov stability theory, we design a new adaptive anti-synchronization controller with a novel parameter update law. With this adaptive controller, one can anti-synchronize the chaotic Lorenz system and the chaotic Chen system effectively and identify the system's parameters accurately.

* Corresponding author.
E-mail address: sawalha_moh@yahoo.com (M. Mossa Al-sawalha).

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The rest of the paper is organized as follows. Section 2 briefly describes the problem formulation and systems description. In Section 3, we present the adaptive anti-synchronization scheme with a parameter update law for two identical Chen systems. Section 4 presents the adaptive anti-synchronization scheme with a parameter update law for two different chaotic systems, i.e., Chen and Lorenz systems. A conclusion is given at the end.

2. Adaptive anti-synchronization

Consider the drive chaotic system in the form of

$$\dot{x} = f(x) + F(x)\alpha \tag{1}$$

where $x \in \Omega_1 \subset R^n$ is the state vector, $\alpha \in R^m$ is the unknown constant parameters vector of the system, $f(x)$ is an $n \times 1$ matrix, $F(x)$ is an $n \times m$ matrix and the elements $F_{ij}(x)$ in matrix $F(x)$ satisfy $F_{ij}(x) \in L_\infty$ for $x \in \Omega_1 \subset R^n$. On the other hand, the response system is assumed by

$$\dot{y} = g(y) + G(y)\beta + u \tag{2}$$

where $y \in \Omega_2 \subset R^n$ is the state vector, $\beta \in R^q$ is the unknown constant parameters vector of the system, $g(y)$ is an $n \times 1$ matrix, $G(y)$ is an $n \times q$ matrix, $u \in R^n$ is control input vector and the elements $G_{ij}(y)$ in matrix $G(y)$ satisfy $G_{ij}(y) \in L_\infty$ for $y \in \Omega_2 \subset R^n$.

Let $e = y + x$ be the anti-synchronization error vector. Our goal is to design a controller u such that the trajectory of the response system (2) with initial condition y_0 can asymptotically approach the drive system (1) with initial condition x_0 and finally implement the anti-synchronization such that,

$$\lim_{t\to\infty} \|e\| = \lim_{t\to\infty} \|y(t, y_0) + x(t, x_0)\| = 0 \tag{3}$$

where $\|.\|$ is the Euclidean norm.

2.1. Adaptive anti-synchronization controller design

Theorem 1. *If the nonlinear control is selected as*

$$u = -f(x) - F(x)\hat{\alpha} - g(y) - G(y)\hat{\beta} - ke, \tag{4}$$

and adaptive laws of parameters are taken as

$$\begin{aligned} \dot{\hat{\alpha}} &= [F(x)]^T e, \\ \dot{\hat{\beta}} &= [G(y)]^T e, \end{aligned} \tag{5}$$

then the response system (2) can anti-synchronize the drive system (1) globally and asymptotically, where $k > 0$ is a constant, $\hat{\alpha}$ and $\hat{\beta}$ are, respectively, estimations of the unknown parameters α and β where α and β are constants.

Proof. From Eqs. (1)–(2), we get the error dynamical system as follows

$$\dot{e} = F(x)(\alpha - \hat{\alpha}) + G(y)(\beta - \hat{\beta}) - ke. \tag{6}$$

Let $\tilde{\alpha} = \alpha - \hat{\alpha}$, $\tilde{\beta} = \beta - \hat{\beta}$. If a Lyapunov function candidate is chosen as

$$V(e, \tilde{\alpha}, \tilde{\beta}) = \frac{1}{2}\left[e^T e + (\alpha - \hat{\alpha})^T(\alpha - \hat{\alpha}) + \left(\beta - \hat{\beta}\right)^T\left(\beta - \hat{\beta}\right)\right], \tag{7}$$

then the time derivative of V along the trajectory of the error dynamical system (6) is as follows

$$\begin{aligned} \dot{V}(e, \tilde{\alpha}, \tilde{\beta}) &= \dot{e}^T e + (\alpha - \hat{\alpha})^T \dot{\tilde{\alpha}} + \left(\beta - \hat{\beta}\right)^T \dot{\tilde{\beta}} \\ &= [F(x)(\alpha - \hat{\alpha}) + G(y)(\beta - \hat{\beta}) - ke]^T e - (\alpha - \hat{\alpha})^T [F(x)]^T e - (\beta - \hat{\beta})^T [G(y)]^T e \\ &= -ke^T, \quad e < 0, \end{aligned} \tag{8}$$

as long as $e \neq 0$, thus, $\frac{dV}{dt} < 0$ for $V > 0$, and the proof follows from the Theorem of Lyapunov on asymptotic stability. □

Remark 1. Most typical chaotic systems can be described by (1), such as the Lorenz system, the Chen system, the Lü system, the Rössler system, the unified system, the van der Pol oscillator, the Duffing oscillator and several variants of Chuas circuits.

Remark 2. If system (1) and system (2) satisfies $f(\cdot) = g(\cdot)$ and $F(\cdot) = G(\cdot)$, then the structure of system (1) and system (2) is identical. Therefore, Theorem 1 is also applicable to the adaptive anti-synchronization of two identical chaotic systems with unknown parameters.