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A simple way to model the pressure dependency of rock velocity

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ABSTRACT

Modeling the pressure dependency of rock velocity is important for interpreting and comparing the seismic and earthquake data from different depths. This study develops a multicomponent differential effective medium model for the elastic properties of porous rocks with two types of pores in the grain background without mixing order. The developed model is applied to modeling the pressure dependent elastic velocity of porous rocks by incorporating the variation of stiff and compliant porosity as a function of pressure. The pressure dependent stiff and compliant porosity were inverted from the measured total porosity under pressure using a dual porosity model, and the unknown constant stiff and compliant pore aspect ratios were inverted by best fitting the modeled velocity to the measured data. Application of the approach to a low porosity granite and a medium porosity sandstone sample showed that the pressure dependency of rock velocity can be satisfactorily modeled by the developed model using the pressure dependent stiff and compliant porosity and carefully estimated stiff and compliant pore aspect ratio values.

1. Introduction

Elastic velocity of porous rocks increases exponentially with increasing pressure and reaches asymptotic values at high pressures (e.g., Han et al., 1986; Eberhart-Phillips et al., 1989; Zimmerman, 1991; Khaksar et al., 1999; King, 2005; Agersborg et al., 2008; Han et al., 2011). The pressure dependency of rock velocity can be attributed to the mineralogical composition and porosity of the rock, the properties of the fluid saturating the pores, and is also greatly controlled by the microstructure of the pore space (David and Zimmerman, 2012). While porosity and properties of the minerals and the pore fluid can be easily determined through laboratory measurements, it is generally difficult to quantify the complex pore structure. Therefore, the pore structure is usually simplified to model its effect on the pressure dependency of elastic velocity.

In addition to the models that try to simply describe the complicated pore structure as a whole in terms of crack density, defined as the number of cracks per unit volume times the crack radius cubed (e.g., Sayers, 2002; Grechka and Kachanov, 2006; Vernik and Kachanov, 2010), there are mainly two kinds of simplification made to describe the pore structure. The first one idealizes the shape of the pores by assuming that they can be represented by spheroids with a specific aspect ratio α. The pore structure is then simplified to be composed of a finite or complete distribution of pores with different aspect ratios, which can be obtained by inversion of velocity measured as a function of pressure (Cheng and Toksöz, 1979; Fortin et al., 2007; Adelinet et al., 2011; David and Zimmerman, 2012). The idealization of pores as spheroids is recognized as a good assumption, because spheroids (1) can capture some essential properties of the subsurface voids, (2) can provide intuitively simple parameterization of enormous complexity of the real pore space and (3) are relatively easily amenable to theoretical analysis (Gurevich et al., 2009a). However, these simplified models are still complicated in terms of their large number of unknowns, and the inverted pore aspect ratio varies between the different effective medium models employed to link the measured elastic velocity to the microstructure of the rock.

The second kind of simplification assumes that the pore space of rocks has a binary structure (e.g., Mavko and Jizba, 1991; Shapiro, 2003; Gurevich et al., 2009b): stiff pores, which form most of the pore space and contribute mainly for the high-pressure velocity, and a small amount of compliant pores, which are responsible for the exponential increase of elastic velocity at lower pressures. Without assuming a specific shape for the stiff or compliant pores, the dual porosity concept of Shapiro (2003) explains well the pressure dependency of dry rock velocity (Pervukhina et al., 2010). However, the variables in the dual porosity model can only be obtained by fitting the measured both P- and S-wave velocity data, thus limiting its predictive power.

A logical extension of these two partially successful kinds of simplification is to investigate if a further simplification can be made to the pore structure so that the pressure dependency of rock velocity can be more simply modeled. To achieve this, I first present an algorithm to extend the conventional 2-component differential effective medium (DEM) model to accommodate different types of pores (i.e., multicomponent) without ordering effect. I then assume the pore to be composed of stiff and compliant pores each with a constant aspect ratio. The pressure dependency of elastic velocity of

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porous rocks is finally modeled using the developed model by incorporating the pressure dependent stiff and compliant porosity as well as their aspect ratios. The aim of this paper is to investigate whether the proposed model can sufficiently model the measured variation of velocity with pressure using the further simplified pore structure (i.e., stiff and compliant pores with a constant aspect ratio, respectively), rather than demonstrating the predictive power of the model. Once the applicability of the suggested method is confirmed, the pressure dependent velocity of a rock can then be predicted provided that the information of the porosity under stress and the stiff and compliant pore aspect ratios is available.

2. Theoretical models

2.1. Two-component differential effective medium model

In the conventional 2-component differential effective medium (DEM) model, an infinitesimal volume of the inclusion (i.e., the pores) is added to replace the host material (i.e., the rock grains), and the newly constructed effective medium forms the background for the next iteration until the final volume of the inclusion is reached. Berryman (1992) showed that the changes in the bulk (K_{DEM}) and shear (μ_{DEM}) moduli due to an increase in the volume of the inclusion df_i, are given by

$$\begin{aligned} dK_{\mathrm{DEM}}(f_i) &= \frac{(K_i - K_{\mathrm{DEM}})P_i(f_i)}{1-f_i}df_i \\ d\mu_{\mathrm{DEM}}(f_i) &= \frac{(\mu_i - \mu_{\mathrm{DEM}})Q_i(f_i)}{1-f_i}df_i \end{aligned}, \quad (1)$$

where K_i, μ_i and f_i are the bulk and shear moduli and volume fraction of the inclusion, and P_i and Q_i are geometric coefficients for the inclusion in a background medium with an effective bulk modulus K_{DEM} and a shear modulus μ_{DEM}, respectively. The initial conditions for the DEM model are defined as $K_{DEM}(0) = K_1$ and $\mu_{DEM}(0) = \mu_1$, where K_1 and μ_1 are the bulk and shear moduli of the host material, respectively.

2.2. Multicomponent differential effective medium model

One major problem of the DEM model is that for multiple inclusion shapes or multiple constituents, the effective moduli depend not only on the final volume fractions of the constituents but also on the order in which the incremental additions are made (Mavko et al., 2009). To eliminate this problem, this work proposes an algorithm to extend the 2-component DEM model to accommodate two inclusions without ordering effects. The two inclusions can then be used to represent stiff and compliant pores. The algorithm for the new multicomponent DEM model is consistent with the incremental concept employed by Berg (2007) and Han et al. (2015) for electrical conductivity of porous rocks.

In the multicomponent (three-component in this work) DEM model, the second component with a volume fraction of V_2/n (where V_2 is the initial volume of component 2 and n is the total number of iterations) is first added into the grain background (the first component), replacing the same amount of the grain and forming an effective background for the next step (step 2 as shown in Fig. 1) in which component 3 with a volume fraction of V_3/n is then included to replace the effective background and form a new effective medium. The effective medium formed when the two components are included in each iteration becomes a background for the next iteration until the total number of iterations n is achieved. The effective medium formed in each step is assumed to be an effective single component, and the 2-component DEM model (Eq. (1)) is used to calculate the effective elastic moduli by adding the required component with the volume fraction of each component determined as follows.

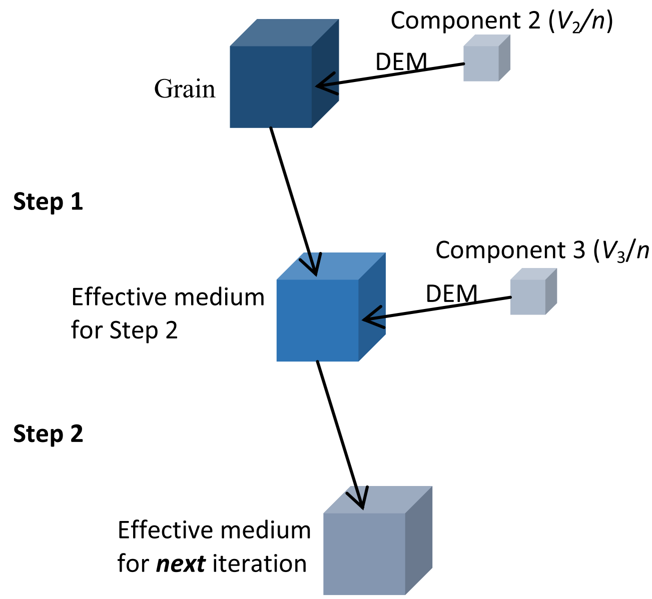

Fig. 1. Schematic diagram showing the steps used in the first iteration of the multicomponent DEM model, in which solid grain is the starting medium. The 2-component DEM is used in each step to add the required inclusion. The effective medium formed in the last step of each iteration becomes the starting background for the next iteration.

In the first iteration, the volume fraction of each component after the second component is included (step 1) is given by

$$\begin{aligned} f_g(1,1) &= 1 - V_2/n, \\ f_2(1,1) &= V_2/n, \\ f_3(1,1) &= 0, \end{aligned} \quad (2)$$

where f_g is the volume fraction of grain and f_2 and f_3 are the volume fraction of component 2 and component 3, respectively. The volume fraction of the constituents after the third component is added (step 2) is calculated as

$$\begin{aligned} f_g(1,2) &= f_g(1,1)(1 - V_3/n), \\ f_2(1,2) &= f_2(1,1)(1 - V_3/n), \\ f_3(1,2) &= V_3/n. \end{aligned} \quad (3)$$

The effective medium formed in the last step of each iteration performs as the starting background for the next iteration, therefore the volume fraction of each component in the i-th iteration ($2 \le i \le n$), after component 2 and component 3 are added is given respectively by

$$\begin{aligned} f_g(i,1) &= f_g(i-1,2)(1 - V_2/n), \\ f_2(i,1) &= f_2(i-1,2)(1 - V_2/n) + V_2/n, \\ f_3(i,1) &= f_3(i-1,2)(1 - V_2/n), \end{aligned} \quad (4)$$

$$\begin{aligned} f_g(i,2) &= f_g(i,1)(1 - V_3/n), \\ f_2(i,2) &= f_2(i,1)(1 - V_3/n), \\ f_3(i,2) &= f_3(i,1)(1 - V_3/n) + V_3/n. \end{aligned} \quad (5)$$

The sum of the volume fraction of each component in each step of any iteration satisfies$f_g(i,j) + f_2(i,j) + f_3(i,j) = 1$, where $i = 1{:}n$, and $j = 1{:}2$.

For a rock with a known volume fraction (f) of the components (e.g., from laboratory measurements), the initial volume (V) of each component for the model input can be determined from Eqs. (2)–(5), based on the assigned number of iterations n. The P- and S-wave velocities of the final effective medium can then be calculated based on the obtained bulk and shear moduli (K and μ, respectively) as $V_p =$