Tectonophysics 675 (2016) 181–195

Contents lists available at ScienceDirect

Tectonophysics

journal homepage: www.elsevier.com/locate/tecto

Adaptive regularization of earthquake slip distribution inversion

Chisheng Wang [a,b], Xiaoli Ding [c], Qingquan Li [a,*], Xinjian Shan [d], Jiasong Zhu [a], Bo Guo [a], Peng Liu [a]

[a] *Shenzhen Key Laboratory of Spatial Smart Sensing and Services, Key Laboratory for Geo-Environment Monitoring of Coastal Zone of the National Administration of Surveying, Mapping and GeoInformation, Shenzhen University, Shenzhen, Guangdong, China*
[b] *College of Information Engineering, Shenzhen University, Shenzhen, Guangdong, China*
[c] *The Department of Land Surveying and Geo-Informatics, The Hong Kong Polytechnic University, Hung Hom, Kowloon, Hong Kong, China*
[d] *State Key Laboratory of Earthquake Dynamics, Institute of Geology, China Earthquake Administration, Beijing, China*

ARTICLE INFO

Article history:
Received 23 June 2014
Received in revised form 9 March 2016
Accepted 14 March 2016
Available online 21 March 2016

Keywords:
Adaptive regularization
Weighted total least squares
Earthquake
Slip distribution
Inversion

ABSTRACT

Regularization is a routine approach used in earthquake slip distribution inversion to avoid numerically abnormal solutions. To date, most slip inversion studies have imposed uniform regularization on all the fault patches. However, adaptive regularization, where each retrieved parameter is regularized differently, has exhibited better performances in other research fields such as image restoration. In this paper, we implement an investigation into adaptive regularization for earthquake slip distribution inversion. It is found that adaptive regularization can achieve a significantly smaller mean square error (MSE) than uniform regularization, if it is set properly. We propose an adaptive regularization method based on weighted total least squares (WTLS). This approach assumes that errors exist in both the regularization matrix and observation, and an iterative algorithm is used to solve the solution. A weight coefficient is used to balance the regularization matrix residual and the observation residual. An experiment using four slip patterns was carried out to validate the proposed method. The results show that the proposed regularization method can derive a smaller MSE than uniform regularization and resolution-based adaptive regularization, and the improvement in MSE is more significant for slip patterns with low-resolution slip patches. In this paper, we apply the proposed regularization method to study the slip distribution of the 2011 Mw 9.0 Tohoku earthquake. The retrieved slip distribution is less smooth and more detailed than the one retrieved with the uniform regularization method, and is closer to the existing slip model from joint inversion of the geodetic and seismic data.

1. Introduction

Earthquake slip inversion with geodetic constraints is an important step in an earthquake study. Many earthquake research fields rely on its results, including earthquake mechanics (Dmowska et al., 1996), recurrence estimation (Thatcher, 1990), earthquake triggering (Lin and Stein, 2004), and hazard assessment studies (Nishimura et al., 2000). Modern geodesy, especially GPS and InSAR (Zhang et al., 2014), have greatly increased the number of constraints on inversion (Wang et al., 2012; Massonnet et al., 1994; Johnson et al., 2001), but using finite observations to invert the infinite fault slip on a continuous plane is always an ill-posed problem. Normally, discretization is first applied to the fault plane so that the infinite number of parameters can be reduced to a finite number of parameters. However, even when the number of parameters is less than the number of observations, the inverse problem often remains ill-posed because there are always poorly constrained slip patches located at depth or in areas sparsely covered by measurements.

Therefore, regularization is necessary to avoid numerically abnormal solutions of the ill-posed slip inversion problem. The widely applied regularization methods, such as truncated singular value decomposition (TSVD) (Hansen, 1987) and the Tikhonov method (Tikhonov et al., 1977), have also been applied to earthquake slip distribution in previous works (Fornaro et al., 2012; Pritchard et al., 2002). Please note that in the Bayesian inversion approaches (Jolivet et al., 2014; Ide et al., 1996; Minson et al., 2013), the regularization normally corresponds to the prior probability. The application of the regularization in slip distribution inversion is mostly uniform-type regularization (Wright et al., 2004; Jónsson et al., 2002), where all the slip patches are treated uniformly with the same regularization. Little attention has been paid to adaptive regularization (Lohman, 2004), even though it gives better performances in many fields such as image restoration (Kang and Katsaggelos, 1995). Alternatively, some research interest has been paid to varying the grid size on the fault plane, where the fault is divided based on the resolution matrix (Simons et al., 2002; Page et al., 2009; Barnhart and Lohman, 2010; Atzori and Antonioli, 2011).

This paper aims to explore the adaptive regularization of slip distribution inversion. We start with an analysis of some widely accepted regularization methods and an investigation into the relationship between discretization and regularization. We then examine the

* Corresponding author.
E-mail address: liqq@szu.edu.cn (Q. Li).

http://dx.doi.org/10.1016/j.tecto.2016.03.018
0040-1951/

performance of the existing non-uniform fault discretization method and some other kinds of adaptive regularization methods. Finally, we propose a novel adaptive regularization method based on weighted total least squares (WTLS). To validate the performance of the new method, the results of a series of simulation experiments and a case study of the 2011 Tohoku earthquake are presented.

2. Regularization of slip distribution inversion

2.1. The ill-posed slip distribution inversion problem

Fault slip can occur in every point of a fault plane, and should therefore be described by a continuous function $m(\boldsymbol{x})$ with the coordinates $(\boldsymbol{x})$ as variables. To invert the fault slip from the geodetic constraints is a standard continuous inverse problem, which can be described by a Fredholm integral equation of the first kind (Menke, 2012):

$$d_i = \int_V G_i(\boldsymbol{x}) m(\boldsymbol{x}) d\boldsymbol{x}, (i = 1, 2\ldots, N) \tag{1}$$

where d_i is the ith observed displacement, G is the elastic response of the earth (Green's functions), m is the fault slip, and V is the volume of the coordinates on the fault plane.

Because the solution to Eq. (1) is not unique, we need to first parameterize the continuous function $m(\boldsymbol{x})$ by a finite number (M) of coefficients:

$$m(\boldsymbol{x}) \approx \sum_{j=1}^{M} m_j f_j(\boldsymbol{x}) \tag{2}$$

where $f_j(\boldsymbol{x})$ defines the a priori knowledge about the behavior of a fault slip (Menke, 2012). It is normally assumed that the fault slip is constant in a certain patch. Therefore, $f_j(\boldsymbol{x})$ is selected as a boxcar function, which is unity for the coordinates located on the jth patch and zero for those outside. The coefficient m_j then represents the slip on the jth patch. Other choices of $f_j(\boldsymbol{x})$, such as polynomial approximation, a truncated Fourier series, and splines (Fukahata and Wright, 2008), can also be used to represent $m(\boldsymbol{x})$.

In combining Eqs. (1) and (2), the continuous inverse problem turns out to be a discrete inverse problem:

$$\mathrm{d}_i = \sum_{j=1}^{M} \mathrm{G}_{ij} \mathrm{m}_j \tag{3}$$

where $\mathrm{G}_{ij} = \int_{V_j} G_i(\boldsymbol{x}) f_j(\boldsymbol{x}) d\boldsymbol{x}$. Here, V_j denotes the volume of the coordinates on the jth fault patch. The size of the grid is controlled by prior assumption of the smoothness of the model (Menke, 2012). A smoother slip distribution means that the fault slips can be approximately constant in a larger grid.

There are three factors defining a well-posed discrete inverse problem: 1) the existence of a solution; 2) the uniqueness of the solution; and 3) the stability of the solution. With the development of synthetic aperture radar (SAR) satellites and GPS networks, the number of measurements is often larger than the number of parameters (slips on fault patches), and the solution can be uniquely determined using the ordinary least squares method. However, there will still be some poorly constrained fault slips located at depth or in areas sparsely covered by measurements. A small disturbance in the observations can lead to a great disturbance for these slips, and so the uniquely determined solution is unstable. Therefore, the discrete inverse problem shown in Eq. (3) is often ill-posed.

We can apply singular value decomposition (SVD) to matrix $\mathbf{G}$ to analyze how the solution instability comes out:

$$\mathbf{G}_{N\times M} = \mathbf{U}_{N\times N} \mathbf{S}_{N\times M} \mathbf{V}_{M\times M}^{\boldsymbol{T}} \tag{4}$$

where $\mathbf{U}$ and $\mathbf{V}$ are orthogonal matrices and $\mathbf{S}$ is a diagonal matrix. Its elements $\lambda_i (i=1,2\ldots,M)$ are defined as singular values. In using the column vector $\mathbf{u}_i \in R^{N\times 1}$ and $\mathbf{v}_i \in R^{M\times 1}$ to represent the matrices $\mathbf{U}$ and $\mathbf{V}$, the solution $\mathbf{m}$ from the ordinary least squares method is given by:

$$\mathbf{m} = \sum_{i=1}^{M} \frac{\mathbf{u}_i^T \boldsymbol{d}}{\lambda_i} \mathbf{v}_i. \tag{5}$$

Assuming that the real observation $\tilde{\boldsymbol{d}}$ is perturbed from the real value $\boldsymbol{d}$ with noise $\boldsymbol{e}$, the solution $\tilde{\mathbf{m}}$ will take the form of:

$$\tilde{\mathbf{m}} = \boldsymbol{m} + \sum_{i=1}^{M} \frac{\mathbf{u}_i^T \boldsymbol{e}}{\lambda_i} \mathbf{v}_i. \tag{6}$$

For an ill-posed inverse problem, λ_i decreases quickly from $\mathrm{i} = 1$ to $\mathrm{i} = M$. Fig. 1 shows an example of the decreasing λ_i from an ill-posed inverse problem, which uses 2939 InSAR observations to solve the slips on 120 fault patches. We can see that λ_i is almost zero when i is larger than 40. This indicates that the noise part $\frac{\mathbf{u}_i^T \boldsymbol{e}}{\lambda_i} \mathbf{v}_i$ in the solution $\tilde{\mathbf{m}}$ is very high.

The purpose of regularization is to find a filter factor τ_i to suppress the instability caused by the small λ_i, and not to affect the large λ_i significantly at the same time. The solution with a filter factor τ_i is given by:

$$\tilde{\mathbf{m}} = \sum_{i=1}^{M} \tau_i \frac{\mathbf{u}_i^T \boldsymbol{d}}{\lambda_i} \mathbf{v}_i + \sum_{i=1}^{M} \tau_i \frac{\mathbf{u}_i^T \boldsymbol{e}}{\lambda_i} \mathbf{v}_i. \tag{7}$$

The difference between regularization methods lies in the choice of filter factor τ_i.

2.2. Regularization methods

Before introducing the regularization methods, we firstly define the metric to evaluate the regularized solution. After regularization, the solution $\tilde{\mathbf{m}}$ becomes a biased solution, which means the expectation of $\tilde{\mathbf{m}}$ is not equal to the true solution $\boldsymbol{m}$. Three terms are involved in describing the accuracy of $\tilde{\mathbf{m}}$, including bias ($\mathbf{B}$), solution variance ($\mathbf{D}$) and mean square error (MSE). Bias ($\mathbf{B}$) denotes the discrepancy between the expectation of $\tilde{\mathbf{m}}$ and true value $\boldsymbol{m}$. Solution variance

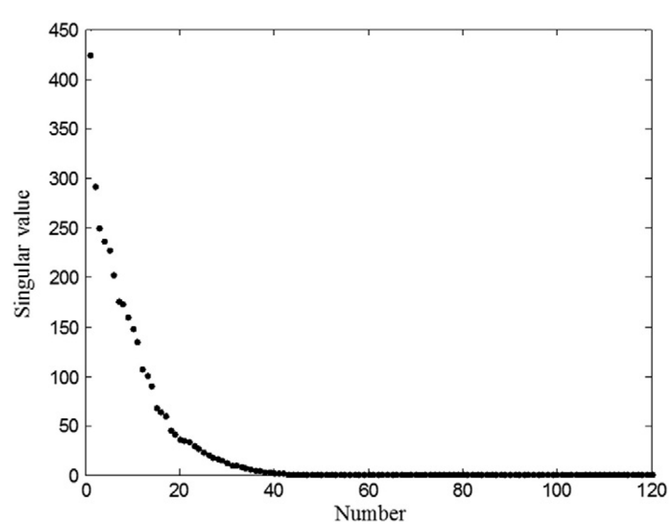

Fig. 1. Singular values from an ill-posed inverse problem.