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Shape fabric development in rigid clast populations under pure shear: The influence of no-slip versus slip boundary conditions

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ABSTRACT

Shape fabrics of elliptical objects in rocks are usually assumed to develop by passive behavior of inclusions with respect to the surrounding material leading to shape-based strain analysis methods belonging to the R_f/ϕ family. A probability density function is derived for the orientational characteristics of populations of rigid ellipses deforming in a pure shear 2D deformation with both no-slip and slip boundary conditions. Using maximum likelihood a numerical method is developed for estimating finite strain in natural populations deforming for both mechanisms. Application to a natural example indicates the importance of the slip mechanism in explaining clast shape fabrics in deformed sediments.

1. Introduction

The characterization of crustal deformation in geological studies is typically achieved using strain analysis techniques that use populations of ubiquitous objects found in deformed rocks (e.g., sedimentary clasts) assumed to act as idealized ellipsoidal markers for the purpose of quantifying strain (Ramsay, 1967; Shimamoto and Ikeda, 1976; Robin, 1977). Nearly all existing methodologies make the critical assumption that these ellipsoidal markers acted passively during deformation, i.e., the marker and surrounding rock matrix respond to deformation identically. If the assumption of passive behavior is to be strictly adhered to and respected, the number of valid natural strain markers available for structural studies is severely limited due to the variability in marker/matrix competence contrasts in real rocks. Such heterogeneity is particularly evident when studying deformation in sedimentary rocks where competence contrasts between clasts and matrix can be quite significant. This potential contrast in viscosity between clasts and matrix, especially in conglomerates, has been long recognized (Ramsay, 1967; Gay, 1968a,b; Meere et al., 2008). A number of studies have provided a theoretical treatment of the deformation of objects within a matrix with variable contrast in viscosity (Gay, 1968a,b; Ghosh and Sengupta, 1973; Bilby et al., 1975; Lisle, 1983; Mulchrone and Walsh, 2006; Treagus and Treagus, 2001, 2002). An extreme case of such rheological contrast is one where the clasts behave as rigid objects. An increasing body of analog and numerical modeling studies exist (Ildefonse et al., 1992a,b; Ildefonse and Mancktelow, 1993; Arbaret et al., 1996; Jezek et al., 1996, 1999; Piazolo et al., 2002; Mulchrone, 2007a) that characterize deformation by rigid body rotation of clasts in a weak matrix (Jeffery, 1922). This behavior is increasingly recognized in deformed rocks and sediment where the matrix supporting rigid clasts is mechanically weak, e.g., deformation in glacial tills and fault gouge (Dowdeswell and Sharp, 1986; Hart, 1994; Clark, 1997; Hooyer and Iverson, 2000; Carr and Rose, 2003; Evans et al., 2006; Thomason and Iverson, 2006; Iverson et al., 2007, 2008; Benn and Evans, 2010). Hooyer and Iverson (2000) in an experimental study on clast fabrics in glacial tills were the first to recognize the tendency of clasts during simple shear to rotate into the shear plane and remain there, contrary to what would be expected from the rigid model (Jeffery, 1922) where clasts rotate through the shear plane with ongoing deformation. They attributed this behavior to the likelihood of mechanical 'slip' between clast and matrix during deformation. Meere et al. (2008) recognized a significant component of 'non passive' rigid clast behavior in tectonically deformed sandstones and conglomerates from SW Ireland and western Montana. This study will further explore the potential contribution of clast /matrix slip has to the development of fabrics in rocks where rigid body rotation is the dominant deformation mechanism.

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2. Rigid object rotation under pure shear with stick and slip boundary conditions

Equations to describe the motion of an isolated rigid elliptical inclusion immersed in a very viscous linear fluid with no-slip boundary conditions are well known (Jeffery, 1922). More recently these equations have been extended in 2D to cover the case of a deformable inclusion with no-slip (Mulchrone and Walsh, 2006) and a rigid inclusion with a slip boundary condition (Mulchrone, 2007b). Here, the case of fabric evolution during a pure shear deformation is considered and relevant equations are briefly presented.

The velocity gradient tensor (L) corresponding to incompressible pure shear is:

$$\mathrm{L} = \begin{bmatrix} L_{11} & 0 \\ 0 & -L_{11} \end{bmatrix} \tag{1}$$

where $L_{11} > 0$ results in extension along the x-direction and compression along the y-direction (see Fig. 1). Ordinary differential equations describing the motion of a linear viscous deformable inclusion in 2D in this deformation regime (Mulchrone and Walsh, 2006) with no-slip at the boundary are given by:

$$\frac{d\phi}{dt} = -\frac{L_{11}(R+1)\left(1+2R(\mu_r-1)+R^2\right)\sin 2\phi}{(R-1)\left(1+2\mu_r R+R^2\right)} \tag{2}$$

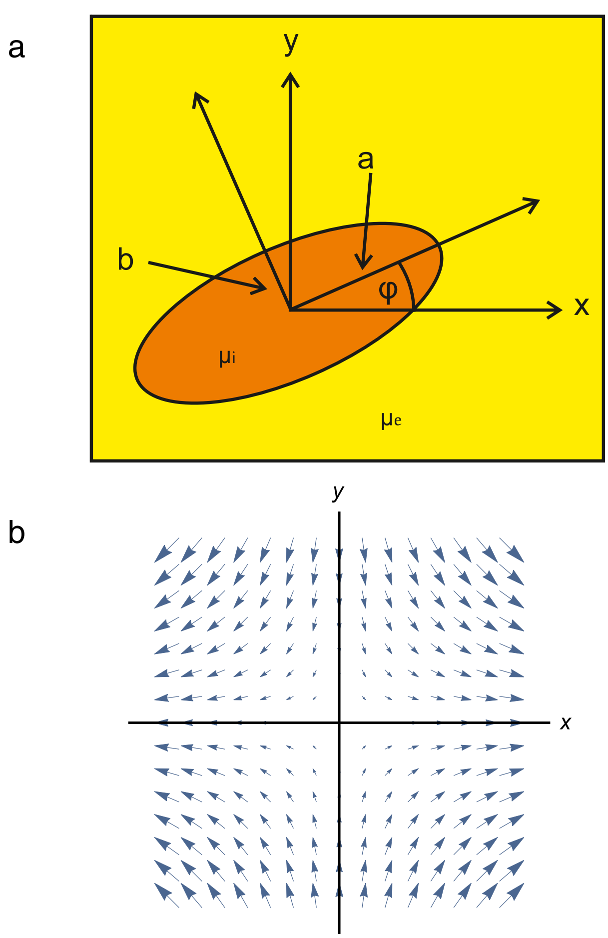

Fig. 1. (a) Definition diagram for elliptical inclusion with long axis of half length a, short axis of half length b and axial ratio $R = \frac{a}{b}$. Long axis makes an angle ϕ with the positive $x-$ direction. Material inside the ellipse has viscosity μ_i and material outside the ellipse has viscosity μ_e. (b) Flow field corresponding to velocity gradient tensor ($\mathbf{L}$) given in Eq. (1). If $L_{11} > 0$ then maximum extension is along the $x-$ direction and maximum compression is along the $y-$ direction.

$$\frac{dR}{dt} = \frac{2L_{11}\mu_r R(1+R)^2\cos 2\phi}{\mu_r\left(1+R^2\right)+2R} \tag{3}$$

where t is time, μ_r is the ratio of the external to the internal viscosity, R is a axial ratio of the inclusion and ϕ is the orientation of the long axis such that the positive x-direction is the zero $\phi-$ direction. Choosing the kinematic framework such that maximum extension is along the x-direction means that it parallels the zero $\phi-$ direction. Given that fabrics tend to parallel the direction of maximum extension, measuring natural inclusions relative to a cleavage is convenient and simplifies the mathematics.

Setting $\mu_r = 1$ Eqs. (2) and (3) gives the motion of a passive inclusion:

$$\frac{d\phi}{dt} = -\frac{L_{11}\left(1+R^2\right)\sin 2\phi}{R^2-1} \tag{4}$$

$$\frac{dR}{dt} = 2L_{11}R\cos 2\phi. \tag{5}$$

Taking initial conditions $R(0) = 1$ and $\phi(0) = 0$ and solving Eqs. (4) and (5), the evolution of the strain ellipse is found to be:

$$R_s = e^{2L_{11}t} \tag{6}$$

which allows time to be parameterized in terms of the axial ratio of the finite strain ellipse (R_s):

$$t = \frac{\ln(R_s)}{2L_{11}}. \tag{7}$$

R_s is a more natural parameter to use in discussing fabric evolution.

Setting $\mu_r = 0$ in Eqs. (2) and (3) the motion of a rigid inclusion with no-slip (Jeffery, 1922) is governed by:

$$\frac{d\phi}{dt} = -\frac{L_{11}\left(R^2-1\right)\sin 2\phi}{R^2+1} \tag{8}$$

and because the inclusion is rigid $\frac{dR}{dt} = 0$. This is referred to as the case of rigid no-slip. The corresponding motion of a rigid inclusion with a slip boundary condition is given by (Mulchrone, 2007a,b):

$$\frac{d\phi}{dt} = -\frac{L_{11}(R+1)\sin 2\phi}{R-1} \tag{9}$$

once again $\frac{dR}{dt} = 0$. This is referred to as the case of rigid slip. Eqs. (8) and (9) are the key equations used in the analysis below and the cases are termed (i) rigid no-slip and (ii) rigid slip respectively.

3. Fabrics for rigid object populations

3.1. Introduction

In this section two probability density functions (pdfs) to describe the fabrics developed by populations of rigid elliptical objects under a pure shear deformation are derived for the cases of (i) rigid no-slip and (ii) rigid slip. Preferred orientations of populations of rigid objects with no-slip (Fernandez et al, 1983; Fernandez, 1987; Jezek et al., 1996; Masuda et al., 1995; Marques and Coelho, 2003) have been studied in the context of a general shear. Here, attention is restricted to the pure shear case along with rigid behavior with slip (Mulchrone, 2007b) and the question of how to estimate associated finite strain. This involves derviation of probability density functions.

A pdf is a function which gives the relative probability that an elliptical object is oriented with its long axis along a particular direction. For