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# Multifractal images of the seismicity in the Ibero-Maghrebian region (westernmost boundary between the Eurasian and African plates)

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The earthquake occurrence has a scale-invariant behavior that can be analyzed by means of a set of non-integer dimensions. The capacity dimension is related to the *number* of spatially distributed earthquakes, that is, to what extent the data space is full; while the dimensions of entropy and correlation are related to the *mode* in which the data fill the space, i.e. how data are ordered and how data are clustered, respectively. The behavior of the multifractal spectrum is asymptotic at the ends of its variation range. The difference between the extreme values of the spectrum, called multifractal spectrum slope, is used to investigate how the earthquakes and its energy are spatially distributed. In this paper we explain the fractal geometry of the seismicity in the Ibero-Maghrebian region, which is located in the westernmost end of the contact between the Eurasian and African plates. This region presents an unevenly spatially distributed seismic activity with areas of strong earthquakes beside others where the seismicity is weaker. The present-day stress field, the fault systems and the characteristic seismicity of the zone make the contact between the Eurasian and African plates an open problem. To address this issue, we have mapped the spatial variation of the fractal dimensions of capacity, entropy, correlation and the multifractal spectrum slope, which are calculated from positions and energies of earthquakes with focal depth between 0 and 120 km. The multifractal approach allows shedding light on the filling, ordination and clustering of the spatially distributed earthquakes and the released energy. The existence of lithospheric faults in response to the strong interaction between plate edges seems to be the cause of the observed shallow and intermediate seismicity and the rupture of the plate boundary in the Betic–Rif–Alboran area.



## 1. Introduction

Classically, a quantitative approach to the seismicity analysis is made through the seismic parameters $a$ and $b$ involved in the Gutenberg–Richter magnitude–frequency relationship. As is well known, the $b$-value expresses the existing proportion between weak and strong events, and an increase in this value means a larger number of weak events against a decrease of the stronger ones, while a smaller $b$-value means the contrary, i.e. a greater number of strong events versus a decrease of the weaker ones. The spatial variation of the $b$-value, which varies in the range 0.6–1.4 being its most common value very close to unity, is related to crack density (heterogeneity) of the materials (Mogi, 1962), stress and strain distributions (Mogi, 1967; Scholz, 1968), geological complexity (López Casado et al., 1995) and deformation velocity of the medium (Manakou and Tsapanos, 2000). The Gutenberg–Richter law is not the only power law for seismicity. Omori (1894) proposed an inverse power law for the distribution of the number of aftershocks with time, which was updated later by Utsu (1961) and Utsu et al. (1995) through a decay law of aftershock activity of the type $(c + t)^{-p}$, where the exponent $p$ takes values in the range 0.7–1.4 and is interpreted in terms of the heterogeneity of the medium ($c$ is a temporal compensation parameter). The number of aftershocks decreases more rapidly for a homogeneous material (high $p$-values) than in the case of a heterogeneous material (low $p$-values).

That said, the fractal geometry enables an alternative approach to the seismicity (Legrand, 2002; Telesca et al., 2003; Turcotte, 1997) as confirmed by mono- and multi-fractal investigation of scaling properties in temporal patterns of seismic sequences (Telesca et al., 2004). The earthquake occurrence in space and time obeys power laws which can be studied through fractal geometry. Several studies have highlighted that many natural phenomena, such as the spatial distribution of the earthquakes, have a scale-invariant behavior and in this sense are fractal objects that present fractal geometry (Legrand, 2002; Mandelbrot, 1989; Telesca et al., 2003, 2004). Fractal objects can be defined as those for which the Hausdorff dimension (Mandelbrot, 1982) is smaller than the topological dimension of the space that contain them

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(Vicsek, 1992). The fractal dimension $D$ is used to elaborate a representation of the fractal set as a whole. This dimension $D$ indicates how much space a fractal set fills. In other words: the more objects has a fractal set, the bigger its fractal dimension. It is a way to estimate the size of the irregularities or the heterogeneity degree of a fractal set when zoomed to finer scales. The fractal $D$ dimension provides information about the geometrical properties of the set depicted.

Now, there are many ways to define the fractal dimension of a certain set, though different theoretical approaches lead to different values of $D$ and consequently to different geometrical properties (Telesca et al., 2004). Therefore, when calculating the $D$-value, it is necessary to establish the definition upon which the calculation is based. On the other hand, fractal sets can be homogeneous or heterogeneous, that is, uniform and symmetric or non-uniform and asymmetric, respectively. A mono-fractal set is homogeneous in terms of scale properties and its description requires a single dimension (Stanley et al., 1999). But if the description of the scale properties of the set needs more than one exponent $D$ of the scaling radius, then the set is called multifractal. There are a plenty of multifractal phenomena in the nature (Mandelbrot, 1989). Given that in these cases the fractal geometry of the set requires the use of more than one fractal dimension, i.e. more than one non-integer dimension $D_q$ (with integer $q$ varying from $-\infty$ to $+\infty$), such fractal sets are called multifractals (Halsey et al., 1986; Hentschel and Procraccia, 1983) and the dimensions $D_q$ are known as multifractal spectrum. The integer number $q \rightarrow +\infty$ provides a greater measure $D_q$ of the fractal set (positions or energies of earthquakes), while $q \rightarrow -\infty$ gives a smaller measure (Martínez López et al., 2001).

The behavior of any physical system is often determined, at least partially, by the spatial distribution of a certain scalar magnitude, such as the particle concentration, the voltage, and the seismic energy. Spatially stationary distributions are called fractals or multifractals depending on what was said above. They are related to the study of the distribution of a physical magnitude on a geometric surface. This surface containing the fractal measure can be an ordinary plane, the surface of a sphere, etc. Normally, a multifractal measure has an infinite number of singularities, so that the surface can be represented as a series of cross-linked fractal subsets whose respective fractal dimensions depend on the singularities that define them, so we talk about generalized dimension $D_q$.

In this context, several authors have applied these concepts to the study of a seismically active region (Bayrak and Bayrak, 2012; Henares et al., 2010; Hirabayashi et al., 1992; Legrand, 2002; Öncel and Wilson, 2006; Tang et al., 2012; Teotia, 2000; Teotia and Kumar, 2011), and specifically to earthquakes and seismogenic zones (Bayrak and Bayrak, 2012; Bhattacharya and Kayal, 2003; Singh et al., 2008, 2009), faults (Öncel et al., 2001), or to earthquakes and faults jointly (Henares et al., 2010). The present study addresses the multifractal properties concerning the epicenters and energy of the earthquakes that occurred in and near the westernmost end of the Iberia–Africa plate boundary. This region is subjected to a complex geodynamic process (Buforn et al., 1995, 2004; Gutsher et al., 2002; Henares et al., 2003; Jolivet et al., 2009; Koulali et al., 2011; Mezcua and Rueda, 1997; Pedrera et al., 2011; Stich et al., 2006; Vergés and Fernández, 2012) and therefore is suited for multifractal seismicity analysis. With this purpose we focused on the scale-invariant behavior of the spatial distribution of the regional seismicity, which can be analyzed through a set of generalized dimensions $D_q$ (of non-integer value) and the calculation of the so-called multifractal spectrum, the $D_q$–$q$ curve, that describe its fractal properties. Being the earthquake occurrence of multifractal character as shown by the $D_q$–$q$ curve, dimensions $D_q$ for negative $q$ values report on the empty zones of the fractal set, while for positive $q$ values inform about the clustering of the fractal elements. Once calculated the multifractal spectrum, here we address the fractal dimensions of capacity, entropy, correlation and the multifractal spectrum slope, whose respective spatial variations are analyzed along the Ibero-Maghrebian plate contact.

## 2. The scenario

The study area is located in the westernmost contact zone between the Eurasian and African plates, and it stretches west-to-east between Azores in the Atlantic Ocean and Tunisia in North Africa (Fig. 1).

### 2.1. Contact between plates and fault systems

From Azores to the Gorringe Heights (SW of the St. Vincent Cape) the plate boundary is relatively clear and mainly consists in dextral strike-slip transcurrent faults (reverse faults in some places). In the Gorringe Heights there are some reverse transverse faults in NE–SW direction that cut the contact. In the Horseshoe sector, further down the south coast of Portugal, there are some reverse faults also in NE–SW direction (Rosas et al., 2012). The contact between plates extends to the Gulf of Cadiz; but the complex seismotectonic structure from this place eastward, Gibraltar Arc and Alboran Sea (whose thinned continental crust blurs the plate limit) makes rather difficult to establish its real position, and is possible even more than one seismic alignment. Between Gibraltar and northeastern Morocco exist numerous faults, among them is the major sinistrorse fault system in NE–SW direction. This system comes from northern Europe and stretches from the southeast of Spain to southwest of Morocco (Sanz de Galdeano, 1990a). In the Algeria area we can observe a predominance of reverse faults in direction N70°E.

New fault systems originated in the Betic–Rif region after the Alpine orogeny episode (Sanz de Galdeano et al., 2003, 2005). The whole area comprises several sets of faults (Fig. 1). The longest ones extend in direction N70°E from the Gulf of Cadiz to Alicante. Other faults extend in E–W direction along the Alpujarra corridor and between Malaga and Almeria. The Betic Cordillera also is crossed by other not so long faults. The main fracture lines of the zone are in Nerja and the Granada and Tiscar basins. The Lorca and Murcia faults together with those of Carboneras and Palomares with azimuth N45°E are the most remarkable in the southeast of the Iberian Peninsula. These faults extend up to reach the easternmost end of the Alboran Sea and the Rif Mountains (Hernández et al., 1987). Major faults continue in NE–SW direction up to Agadir (southwest of Morocco, in the Atlantic coast), through Melilla (North African coast) and the Atlas Mountains (Fig. 1). These long faults that continue northward to the Rhone and Rhine basin through the Gulf of Lion (Sanz de Galdeano, 1990a), to the south delimit the Moroccan subplate in the northwestern part of the African Plate (López Casado et al., 2001; Sanz de Galdeano, 1990b). All this makes that the tectonic contact between Iberia and Africa be not as clear as west of the Gulf of Cadiz and east of the Alboran Sea.

### 2.2. Tectonic setting

The Iberian and African plates are undergoing a gradual approximation under a compression regime in NW–SE direction (Fig. 2) (Buforn et al., 1988; Buforn et al., 1995; Udías and Buforn, 1991). In the Atlantic Ocean the approach direction becomes NW–SE (Galindo-Zaldívar et al., 1993; Henares et al., 2003; Udías and Buforn, 1991). In the zones of Algeria and Tunisia the direction of compression is almost north–south (Bezzeghoud and Buforn, 1999; Galindo-Zaldívar et al., 1993; Henares et al., 2003; Medina and Cherkaoui, 1991; Udías and Buforn, 1991). However, in the area of the Betic Cordillera–Alboran Sea an extensional stress regime in NE–SW direction seems to be more plausible than one of compression (Pérez-Peña et al., 2010; Stich et al., 2006), that also would be NE–SW in the southern part of the Alboran Sea and north Morocco (Henares et al., 2003). Other local stresses show north–south compression in the Murcia–Alicante zone (SE of the Iberian Peninsula) (Henares et al., 2003), NW–SE compression and NE–SW extension in the Gulf of Cadiz (Henares et al., 2003; López Casado and Henares, 2011) and NE–SW compression in the Lucena zone, northwest of the