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# Numerical tests on generalized diffraction tomography

Xiaosan Zhu [a,*], Ru-Shan Wu [b], Xiaofei Chen [c], Rui Gao [a]

[a] Institute of Geology, Chinese Academy of Geological Sciences, Beijing 100037, China
[b] Earth and Planetary Sciences Department, University of California, Santa Cruz, CA 95064, United States
[c] School of Earth and Space Sciences, University of Sciences and Technology of China, Herfei 230026, China

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ABSTRACT

In this paper we formulate the generalized diffraction tomography based on the volume scattering model in heterogeneous media. The frequency-dependent effect due to the finite spatial aperture of the acquisition system is corrected in the local angle domain, resulting in less artifacts and more balanced amplitude for recovering velocity perturbations. Using multiple frequencies, the blind area in the spectral domain in the traditional diffraction tomography can be partially filled and the qualities of the local spectra can be improved in both coverage and uniformity in the local wavenumber domain. Through the preliminary tests using a simple box model in a smoothly varying $v(z)$ medium, the generalized diffraction tomography can recover the long-wavelength component of velocity perturbations up to 23% with respect to the background velocity. It can also reconstruct the low-wavenumber-component Marmousi velocity model very well when low-frequency waves are used in the process.



## 1. Introduction

In most cases, inversion theories and methods depend on forward modeling methods. Different modeling methods emphasize different parameters of the models and may obtain quite different inversion results. The traditional diffraction tomography applies a filtered propagation to the scattered field to reconstruct the parameter perturbation based on a homogeneous reference model (Beydoun and Mendes, 1989; Beylkin et al., 1986; Clayton and Stolt, 1981; Devaney, 1982, 1984; Harris, 1987; Ikelle et al., 1986; Lambaré et al., 1992, 2003; Miller et al., 1987; Slaney et al., 1984; Wu and Toksöz, 1987). Through the analysis of travel times of the seismic waves, the traditional tomography focuses on the reconstruction of smooth variations in the velocity model. Devaney (1984) presented the foundation of diffraction tomography for both vertical seismic profiling (VSP) and cross-well tomography. Wu and Toksöz (1987) derived the reconstruction formula of diffraction tomography for the case of surface reflection profiling, the VSP and cross-hole measurements based on the Born or Rytov approximation, which were restricted to weak inhomogeneities.

In order to overcome the limitation of the classic diffraction tomography and obtain more accurate inversion results, formulation of the diffraction tomography in the spatial domain (Woodward, 1992; Woodward et al., 1998, 2008) and nonlinear full waveform inversion (FWI) method (Crase et al., 1990; Forgues and Lambaré, 1997; Gauthier et al., 1986; Jin et al., 1992; Joncour et al., 2011; Liu et al., 2005; Min and Shin, 2006; Operto et al., 2003; Pica et al., 1990; Pratt, 1999; Pratt and Worthington, 1988, 1990; Pratt et al., 1998; Sheng et al., 2006; Tarantola, 1984a, 1984b, 2005; Xu and Lambaré, 2006, 2009) have been developed for heterogeneous background media. Pratt and Worthington (1990) used a nonlinear waveform inversion in the frequency domain to incorporate the rigorous finite-difference modeling technique into the inverse procedure. In order to use the full information content of the recorded wavefield and to reduce the waveform misfit between the observed and the modeled data, Pratt (1999) applied and evaluated a frequency-space domain approach to waveform inversion, which was a local descent algorithm that proceeds from a starting model to refine the model iteratively. However, in these formulation and waveform inversion approaches in spatial domain, the intuitive spectral inversion and its efficiency are lost (Wu, 2007). Hence, some good linear inversion methods, including diffraction tomography, are still desirable, since it can serve as the basis for an efficient nonlinear inversion.

In the new development of generalization of diffraction tomography (Gelius et al., 1991; Wu, 2007) the scattered wave field can be calculated by the distorted-wave Born approximation in heterogeneous media (Gelius et al., 1991; Schultz and Jaggard, 1987; Taylor, 1972). Schultz and Jaggard (1987) used a distorted-wave Born and geometric-optics approximation for the reconstruction algorithm, which is based on a weighted generalized backprojection operator, to establish a connection between the coherent swept-frequency microwave scattering data and the projections of refractive objects. Gelius et al. (1991) also extended

* Corresponding author at: Institute of Geology, Chinese Academy of Geological Sciences, 26 Baiwanzhuang Road, Beijing 100037, China. Tel.: +86 10 13661041092; fax: +86 10 68999729.

*E-mail addresses:* zhuxiaosan129@gmail.com (X. Zhu), wrs@pmc.ucsc.edu (R.-S. Wu), xfchen1@ustc.edu.cn (X. Chen), gaorui@cags.ac.cn (R. Gao).

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diffraction tomography to nonuniform background models using an asymptotic ray theory for the calculation of the background Green's function, which can handle irregular acquisition geometries. However, the ray-approximated Green's function cannot completely handle all the wave phenomena.

Recently, a unified theory was developed for the true-reflection imaging based on the resolution operator of Backus and Gilbert (1967, 1968, 1970), combined with different kinds of forward modeling operators, such as the Born scattering for volume heterogeneities (Wu, 2007; Zhu and Wu, 2009) and boundary scattering for sharp discontinuities. For implementation, we proposed to decompose the operators in the local wavenumber domain and performed the correction therein (Wu, 2007; Zhu and Wu, 2009).

Wu (2007) derived the formulation of scattering tomography in heterogeneous media for the volume scattering. We use a forward scattering renormalized Green's function based on the De Wolf approximation (De Wolf, 1971, 1985; Wu, 1996, 2003; Wu et al., 2007), which is a multiple forward scattering and single back scattering approximation and can be implemented by an iterative marching algorithm (Liu et al., 2007; Wu, 1996, 2003; Wu et al., 2007; Xie et al., 2005).

In this paper, we summarize the theory of generalized diffraction tomography based on the distorted-wave Born model, and present the deconvolution filtering using the local image matrices (LIMs) and their corresponding resolution matrices in the local wavenumber domain, and apply the amplitude correction factors to the amplitude of the wavefield. The results of several numerical tests show excellent resolving capability of the method.

The outline of this paper is the following. Firstly, we briefly review the theory and method of the generalized diffraction tomography, and we describe the distorted-wave Born modeling for the volume scattering model, the imaging condition in the local wavenumber domain, and the local image matrix. We then present the resolving kernel and deconvolution filtering in the local wavenumber domain. Following that, we provide an algorithm for amplitude correction at the dominant frequency for the Born model. We then use several numerical examples to test the spectral recovery and the velocity-anomaly reconstruction. Finally, we give some discussions and conclusions.

## 2. Theory and method of generalized diffraction tomography

The generalized diffraction tomography consists of backpropagation plus filtering in the local wavenumber domain. The backpropagation is a double-focusing operation, which focuses both the wavefields from the source array and the receiver array to the image point. The filtering is a deconvolution in the local wavenumber domain. In this section we describe the theory and formulation of the generalized diffraction tomography (Cao, 2008; Wu, 2007; Zhu and Wu, 2009).

### *2.1. Distorted-wave Born modeling for volume scattering model*

The modeling operator in general is nonlinear to the model parameters, which means the response to the model perturbation at one location will depend on the responses at other locations (multiple scattering). In the true-reflection imaging approach, we try to linearize the problem in an optimal way, which can not only give an optimal reflectivity recovery, but also form a basis for developing efficient iterative inversion methods. In this section we introduce a linearized modeling procedure using the Born model.

In the Born model, the scattering of each volume element is independent from each other and no interactions between elements are taken into consideration. This approximation is valid for weak perturbations and short propagation distances. The parameters to be inverted in the Born model are perturbations of unknown. In the case of scalar media (e.g., acoustic media with a constant density) and the object is described by the velocity distribution $c(\mathbf{x})$ with respect to the background velocity $c_0(\mathbf{x})$, where $\mathbf{x}$ is the position vector, the scattered field under the Born approximation, which is measured by the receiver on the surface ($z = 0$) located at $x_g$ and excited by the source on the surface located at $x_s$, is

$$p^{sc}\left(x_g; x_s\right) = -k^2 \int_V G_F\left(x_g; \mathbf{x}\right) O(\mathbf{x}) G_F(\mathbf{x}; x_s) dv(\mathbf{x}), \tag{1}$$

with

$$O(\mathbf{x}) = 1 - \frac{c_0^2(\mathbf{x})}{c^2(\mathbf{x})}. \tag{2}$$

where $p^{sc}(x_g;x_s)$ is the scattered field; $G_F(\mathbf{x};x_s)$ and $G_F(x_g;\mathbf{x})$ are the scalar wave Green's functions for the modeling process in the inhomogeneous background media. The Green's function $G_F(\mathbf{x};x_s)$ is the wavefield at $\mathbf{x}$ due to a point source on the surface located at $x_s$. Likewise, $G_F(x_g;\mathbf{x})$ is the Green's function for a receiver at $x_g$ due to a source at $\mathbf{x}$; $O(\mathbf{x})$ is the velocity perturbation function of the medium, which is the object function in classic diffraction tomography (Devaney, 1982, 1984; Slaney et al., 1984; Wu and Toksöz, 1987). $k = \omega/c_0(\mathbf{x})$ is the background wavenumber where $\omega$ is the circular frequency and $c_0(\mathbf{x})$ is the background velocity at point $\mathbf{x}$. $V$ is the integral volume of the heterogeneous medium. Hence the modeling operator for the Born model is

$$\mathbf{F}\left(\omega, x_g, x_s | \mathbf{x_0}\right) = -k^2 \int_V G_F\left(x_g; \mathbf{x_0}\right) G_F(\mathbf{x_0}; x_s) dv(\mathbf{x_0}). \tag{3}$$

where $\mathbf{F}(\omega, x_g, x_s|\mathbf{x_0})$ is the modeling operator which maps the model into the data set. We use a forward scattering renormalized Green's function in the model, and the forward modeling is based on the De Wolf approximation (De Wolf, 1971, 1985; Wu, 1996, 2003; Wu and Xie, 2009; Wu et al., 2006, 2007; Xie et al., 2005, 2006), which updates the Green's function by the multiple forward scattering and keeps a single backscattering approximation in the modeling process. The De Wolf approximation is a kind of local Born approximation since the Born approximation applies only locally to the individual thin-slabs (Wu and Xie, 2009; Wu et al., 2008). Hence the model defined by Eq. (1) with the De Wolf approximated Green's function can be called the De Wolf-Born model (or the local Born model) in comparison with the Born model for the classic diffraction tomography in homogeneous background media. It can extend the application to large volume weak perturbations and can overcome some drawbacks and limitations of the Born modeling.

### *2.2. Imaging condition and local image matrix*

Imaging process can be defined mathematically as the backpropagation (or reverse-time propagation) plus focusing (imaging principle) (Claerbout, 1971, 1985). During the process of seismic data, prestack migration is an imaging process with double-focusing operator, which focuses both the waves from the source array and those from the receiver array to the image point. In our method, the double-focusing (imaging) process we used includes the backpropagation of the wavefield and applying an imaging condition at the image point (Claerbout, 1971, 1985). In order to be symmetric for the source array focusing and receiver array focusing and compare with the classic diffraction tomography, we slightly modify the imaging condition from the standard imaging condition in the form of cross-correlation and obtain the formula for the tomographic imaging condition (Cao, 2008; Wu, 2007; Wu and Toksöz, 1987; Zhu and Wu, 2009):

$$I(\mathbf{x}) = \mathrm{Re} \int_{B(\omega)} d\omega f(\omega) L(\omega, \mathbf{x}), \tag{4}$$