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Flexure of the lithosphere and the geodynamical evolution of non-cylindrical rifted passive margins: Results from a numerical model incorporating variable elastic thickness, surface processes and 3D thermal subsidence

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ABSTRACT

We present a new numerical model to calculate the surface deflection of a two-dimensional, yet variable thickness, thin elastic plate. The model is based on a multi-grid, finite difference solution of the fourth-order differential equation that incorporates the terms arising from the non-uniform thickness assumption. The model has been developed to calculate the flexural response of the continental lithosphere subjected to an arbitrary, instantaneous stretching. The flexural model is coupled to (a) a finite element, three dimensional thermal model incorporating the conduction, advection and production terms that allows the computation of the thermal subsidence resulting from the stretching-induced perturbation of the isotherms, assuming that the effective elastic thickness is controlled by the depth to a given isotherm; and (b) a finite difference surface process model that assumes that transport is linearly proportional to slope leading to a second-order, diffusion-type partial differential equation. The model also incorporates the effect of sediment compaction. We present a series of simple benchmarks that demonstrate the accuracy of the model. We also present results of simple 2D and 3D stretching experiments highlighting the importance of 3D flexural effects and the assumed variable elastic thickness on the development of a passive margin and its thermal evolution. Finally, we perform a numerical experiment based on a stretching geometry derived from the present-day geometry of the Western AfricaTransform Margin to predict sediment accumulation patterns and a stratigraphic architecture which we can compare to observations.

1. Introduction

Flexural effects during extension and stretching of the continental lithosphere lead to rift flank uplift and overdeepening of the adjacent rift basin areas (Beaumont, 1982; Braun and Beaumont, 1989; Vening Meinesz, 1950). With a few exceptions (Garcia-Castellanos et al., 2002; Sacek et al., 2012; vanWees and Cloetingh, 1996), most quantitative studies of these flexural effects have been limited to two-dimensional analyses (Braun and Beaumont, 1989; Chéry et al., 1992; Weissel and Karner, 1989). In particular, flexural effects along non-cylindrical passive margin segments, such as commonly observed where large fracture zones intersect the continent, have not been quantitatively assessed. This is true too for the three-dimensional patterns of the ensuing thermal subsidence. Although sophisticated models have been developed to simulate the complex sedimentary architecture associated with tectonic subsidence and/or sea level change (Bitzer and Pflug, 1989; Grandjeon and Joseph, 1999; Li et al., 2004; Salles and Duclaux, 2011), none has so far been properly linked to a three-dimensional flexural and thermal model of the underlying lithosphere to study the complex behavior of this coupled system.

Here, we present a recently developed numerical model, which we called Flex3D, that combines a state-of-the-art solver for the thin elastic plate flexure equation with a surface process model and a three-dimensional model of the thermal evolution of the underlying lithosphere. We demonstrate its usefulness in quantifying the uplift and subsidence patterns associated with rifting and their evolution through time following a rifting event that leads to the formation of a passive continental margin. We highlight the importance of three dimensional effects, focusing on the complex geometry of passive margins in the vicinity of an important 'jog' connecting two linear segments and on the effects of strongly varying elastic thickness across the continent–ocean transition following continental rifting. We demonstrate the enhancement of flexural effects where the margin is not linear. Finally, we show how our predictions can be directly compared to the geometry of a passive margin, off the coast of Guinea.

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2. The flexural model

2.1. Basic equation

The deflection, w, of a thin elastic plate subjected to a vertical load q, an in-plane stress field $[\sigma_{xx},\sigma_{yy},\sigma_{xy}]$ and floating on an inviscid fluid of density ρ_a is governed by the following equation (vanWees and Cloetingh, 1994):

$$\begin{aligned}D\frac{\partial^4 w}{\partial x^4}+D\frac{\partial^4 w}{\partial y^4}+2D\frac{\partial^4 w}{\partial x^2\partial y^2}+2\frac{\partial D}{\partial x}\frac{\partial^3 w}{\partial x^3}+\frac{\partial^2 D}{\partial x^2}\frac{\partial^2 w}{\partial x^2}+2\frac{\partial D}{\partial y}\frac{\partial^3 w}{\partial y^3}\\ +\frac{\partial^2 D}{\partial y^2}\frac{\partial^2 w}{\partial y^2}+\nu\frac{\partial^2 D}{\partial y^2}\frac{\partial^2 w}{\partial x^2}+\nu\frac{\partial^2 D}{\partial x^2}\frac{\partial^2 w}{\partial y^2}+2\frac{\partial D}{\partial x}\frac{\partial^3 w}{\partial x\partial y^2}\\ +2\frac{\partial D}{\partial y}\frac{\partial^3 w}{\partial x^2\partial y}+2(1-\nu)\frac{\partial^2 D}{\partial x\partial y}\frac{\partial^2 w}{\partial x\partial y}=q+\Delta\rho g w+\sigma_{xx}h\frac{\partial^2 w}{\partial x^2}\\ +\sigma_{yy}h\frac{\partial^2 w}{\partial y^2}+2\sigma_{xy}h\frac{\partial^2 w}{\partial x\partial y}\end{aligned} \tag{1}$$

where D is the flexural rigidity, given by:

$$D=\frac{Eh^3}{12(1-\nu^2)}. \tag{2}$$

$\Delta\rho=\rho_a-\rho_w$ (where ρ_w is the density of water assumed to fill the space created by the deflection of the plate), g is the acceleration due to gravity, h is effective elastic thickness (EET), E is Young's modulus and ν is Poisson's ratio. ν is assumed to have a uniform value, but E and h are allowed to vary spatially.

Assuming that the plate thickness and elastic rigidity are uniform, Eq. (1) would reduce to the well-known biharmonic equation (Bodine et al., 1981):

$$D\frac{\partial^4 w}{\partial^4 x}+D\frac{\partial^4 w}{\partial^4 y}+2D\frac{\partial^4 w}{\partial^2 x\partial^2 y}=q+\Delta\rho g w+\sigma_{xx}h\frac{\partial^2 w}{\partial x^2}+\sigma_{yy}h\frac{\partial^2 w}{\partial y^2}+2\sigma_{xy}h\frac{\partial^2 w}{\partial x\partial y} \tag{3}$$

for which there exist analytical solutions corresponding to a variety of boundary conditions (see Watts, 2001, for example). Here we will assume that D and h vary in an arbitrary manner in space and time. Consequently Eq. (1) needs to be solved numerically.

The domain of integration is a rectangular area of dimensions L_x by L_y. Boundary conditions are zero deflection ($w=0$) and zero bending ($\partial^2 w/\partial n^2=0$, where n is the direction normal to the boundary), on all four domain boundaries($x=0$, $x=L_x$, $y=0$, $y=L_y$).

2.2. The load

Our main purpose is to solve Eq. (1) to estimate the deflection of the lithosphere following a rifting event during which the crust of original thickness h_c^0 is thinned by a factor $\delta=h_c^0/h_c$ and the mantle part of the lithosphere of original thickness h_m^0 is thinned by a factor $\beta=h_m^0/h_m$. Following Braun and Beaumont (1989) and Weissel and Karner (1989), the strength profile of the lithosphere can be approximated, to first order, by a strong fiber located at a depth z_n, also called the necking depth, i.e. the depth along which the lithosphere would neck if it was not subjected to isostasy. Under such an assumption, thinning of the crust induces a vertical deflection of amplitude

$$w_\delta=-(1-1/\delta)z_n \tag{4}$$

which generates a vertical, isostatically-driven load, q_δ, of amplitude:

$$q_\delta=(1-1/\delta)z_n\Delta\rho g-(1-1/\delta)h_c(\rho_m-\rho_c)g. \tag{5}$$

Thinning of the lithosphere induces also a thermal load, q_t, of amplitude:

$$q_t=\int_0^{h_c+h_m}\rho(z)g\alpha_v\Delta T(z)dz \tag{6}$$

where α_v is the coefficient of thermal expansion and $\Delta T(z)$ the difference in temperature at depth z before and after the extension. Finally, the loading/unloading associated with sedimentation and erosion processes can be approximated by an additional load, q_s, of amplitude:

$$q_s=-(\rho_s-\rho_w)gz_s \tag{7}$$

where z_s is the accumulated (fully compacted) sediment thickness ($z_s>0$) that replaces the water assumed to fill the gap created by the deflection of the surface (Eq. (1)), or the thickness of eroded material ($z_s<0$).

2.3. Finite difference discretization

Eq. (1) is solved on a two-dimensional rectangular grid with regular spacings Δx and Δy, using the following centered finite difference operators at a point of integer coordinates (i,j):

$$\begin{aligned}
\frac{\partial\psi}{\partial x}\Big|(i,j)&\approx\frac{\psi_{(i+1,j)}-\psi_{(i-1,j)}}{2\Delta x}\\
\frac{\partial^2\psi}{\partial x^2}\Big|(i,j)&\approx\frac{\psi_{(i+1,j)}-2\psi_{(i,j)}+\psi_{(i-1,j)}}{\Delta x^2}\\
\frac{\partial^2\psi}{\partial x\partial y}\Big|(i,j)&\approx\frac{\psi_{(i+1,j+1)}-\psi_{(i-1,j+1)}-\psi_{(i+1,j-1)}+\psi_{(i-1,j-1)}}{4\Delta x\Delta y}\\
\frac{\partial^2\psi}{\partial y^2}\Big|(i,j)&\approx\frac{\psi_{(i,j+1)}-2\psi_{(i,j)}+\psi_{(i,j-1)}}{\Delta y^2}\\
\frac{\partial^3\psi}{\partial x^3}\Big|(i,j)&\approx\frac{\psi_{(i+2,j)}-2\psi_{(i+1,j)}+2\psi_{(i-1,j)}-\psi_{(i-2,j)}}{2\Delta x^3}\\
\frac{\partial^3\psi}{\partial x^2\partial y}\Big|(i,j)&\approx\frac{\psi_{(i+1,j+1)}-2\psi_{(i,j+1)}+\psi_{(i-1,j+1)}-\psi_{(i+1,j-1)}+2\psi_{(i,j-1)}-\psi_{(i-1,j-1)}}{2\Delta x^2\Delta y}\\
\frac{\partial^3\psi}{\partial x\partial y^2}\Big|(i,j)&\approx\frac{\psi_{(i+1,j+1)}-2\psi_{(i+1,j)}+\psi_{(i+1,j-1)}-\psi_{(i-1,j+1)}+2\psi_{(i-1,j)}-\psi_{(i-1,j-1)}}{2\Delta x\Delta y^2}\\
\frac{\partial^3\psi}{\partial y^3}\Big|(i,j)&\approx\frac{\psi_{(i,j+2)}-2\psi_{(i,j+1)}+2\psi_{(i,j-1)}-\psi_{(i-j-2)}}{2\Delta y^3}\\
\frac{\partial^4\psi}{\partial x^4}\Big|(i,j)&\approx\frac{\psi_{(i+2,j)}-4\psi_{(i+1,j)}+6\psi_{(i,j)}-4\psi_{(i-1,j)}+\psi_{(i-2,j)}}{\Delta x^4}\\
\frac{\partial^4\psi}{\partial y^4}\Big|_{(i,j)}&\approx\frac{\psi_{(i,j+2)}-4\psi_{(i,j+1)}+6\psi_{(i,j)}-4\psi_{(i,j-1)}+\psi_{(i,j-2)}}{\Delta y^4}.
\end{aligned} \tag{8}$$

The partial differential equation (Eq. (1)) reduces to a set of coupled linear equations, one for each point (i,j) of the regular grid connecting (i,j) to its 12 closest neighbors which can be expressed symbolically in the following matrix form:

$$\mathbf{AW}=\mathbf{Q} \tag{9}$$

where **A** is a positive definite matrix, **W** is the vector of unknown displacements of length equal to the total number of points on the regular grid and **Q** a load vector of the same length.

2.4. Multigrid solver

In order to solve this large system of algebraic equations, a multi-grid, iterative method (Hackbusch, 1985) is used. This method uses a set of nested grids of resolution $(2^l+1)\times(2^l+1)$ for $l=l_{\min},l_{\max}$ and requires three basic ingredients: a smoothing operator that improves the solution at any given level, l; a prolongation operator that interpolates the residual from a coarse grid (level l) to a fine grid (level $l+1$); and a restriction operator that transfers the information (the residual) from fine to coarse grids. Here we use a Gauss–Siedel iterative scheme for the smoothing operator, a bilinear interpolation for the prolongation operator and its adjunct (transposed) for the restriction operator. We also make use of W-cycles to accelerate the