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# Localised folding in general deformations

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ABSTRACT

One control on the buckling of a layer (or layers) embedded in a weaker matrix is the reaction force exerted by the deforming matrix on the layer. If the system is linear and this force is a linear function of the layer deflection, as for linear elastic and viscous materials, the resulting buckles can be sinusoidal or periodic. However if the system is geometrically nonlinear, as in general non-coaxial deformations, or the matrix material is nonlinear, as for nonlinear elastic, non-Newtonian viscous and plastic materials, the buckling response may be localised so that individual packets of folds form; the resulting fold profile is not sinusoidal. These folds are called *localised folds*. Most natural folds are localised. One view is that irregularity derives solely from initial geometrical perturbations. We explore a different view where the irregular geometry results from a softening material or geometrical nonlinearity without initial perturbations. Localised folds form in a fundamentally different way than the Biot wavelength selection process; the concept of a dominant wavelength does not exist. Folds grow and collapse sequentially rather than grow simultaneously. We discuss the formation of localised folds with recent considerations of constitutive behaviour at geological strain rates for general three-dimensional deformations.


## 1. Introduction

The theory of buckling of a layer, or layers, embedded in another material has been studied in geology since the classical work of Hall (1815) although the concepts involved clearly go back nearly 100 years earlier to the Bernoulli family and to Coulomb and Euler. The folds produced by Hall in layers of cloth (Fig. 1a) are *localised* in the sense that they are not sinusoidal and occur in localised packets. Such irregularity has been widely recognised in the geological literature (Fig. 1, Abbassi and Mancktelow, 1990; Biot et al., 1961; Biot, 1965; Fletcher and Sherwin, 1978; Hudleston and Treagus, 2010; Johnson and Fletcher, 1994; Kocher et al., 2006; Mancktelow, 2001; Mancktelow, 1999; Price and Cosgrove, 1990; Ramberg, 1959; Schmalholz, 2006; Zhang et al., 1996; Zhang et al., 2000). In fact most experimentally produced folds are localised including, as a selection, those produced by Biot et al. (1961), Ghosh (1966), Hudleston (1973), Manz and Wickham (1978), Ramberg (1959), Ramsay (1967, Figs. 3–51, 7–29, 7–35), and Watkinson (1976).

The present paper is concerned with such localised folding behaviour. Most studies of fold irregularity in the geological literature derive from the classical work of Biot in the period 1937–1984, some of which are brought together in Biot (1965). That body of work has had a profound impact on the way in which geologists view the folding process but one should appreciate that it is based on small deflections of thin, inextensible layers and linear stability analyses employing linear constitutive relations and geometries. The results are strictly valid for infinitesimal deflections of layers embedded in linear materials such as linear (Hookean) elastic and/or Newtonian viscous materials undergoing coaxial deformations with no shear stress parallel to the layer, but there is a tendency in the geological literature to extend the results of the linear theory to describe finite non-coaxial deformations of thick, extensible layers with nonlinear matrix materials and nonlinear geometries (see Hudleston and Treagus, 2010). This paper is concerned with nonlinear aspects of buckling theory.

In Hobbs and Ord (2012) we show that one would expect from the linearity of the classical Biot-problem that fold systems developed during coaxial deformations at finite strains would be sinusoidal or perhaps non-sinusoidal but still periodic. That paper also shows that the introduction of nonlinear constitutive relations in the form of nonlinear elasticity or strain-rate viscosity weakening behaviour leads to localised folding. The accompanying paper by Schmalholz and Schmid (2012) extends the examples to multilayered power-law viscous materials. Individual natural fold systems show a range of wavelength to thickness ratios ranging from about 2 to somewhere in the range of 15–20 (Table 1 in Hudleston and Treagus, 2010) and generally they lack periodicity (Fig. 1). This observation is commonly interpreted as an outcome from the Biot theory but representing a relatively flat dispersion relation (Biot et al., 1961) so that the wavelength amplification process is not very efficient at selecting a dominant wavelength and is influenced by the statistics of initial geometrical perturbations. It has been well documented that initial geometrical imperfections of sufficiently large wavelength are capable of inducing an irregular buckling response in linear materials undergoing coaxial deformations with no need to appeal to

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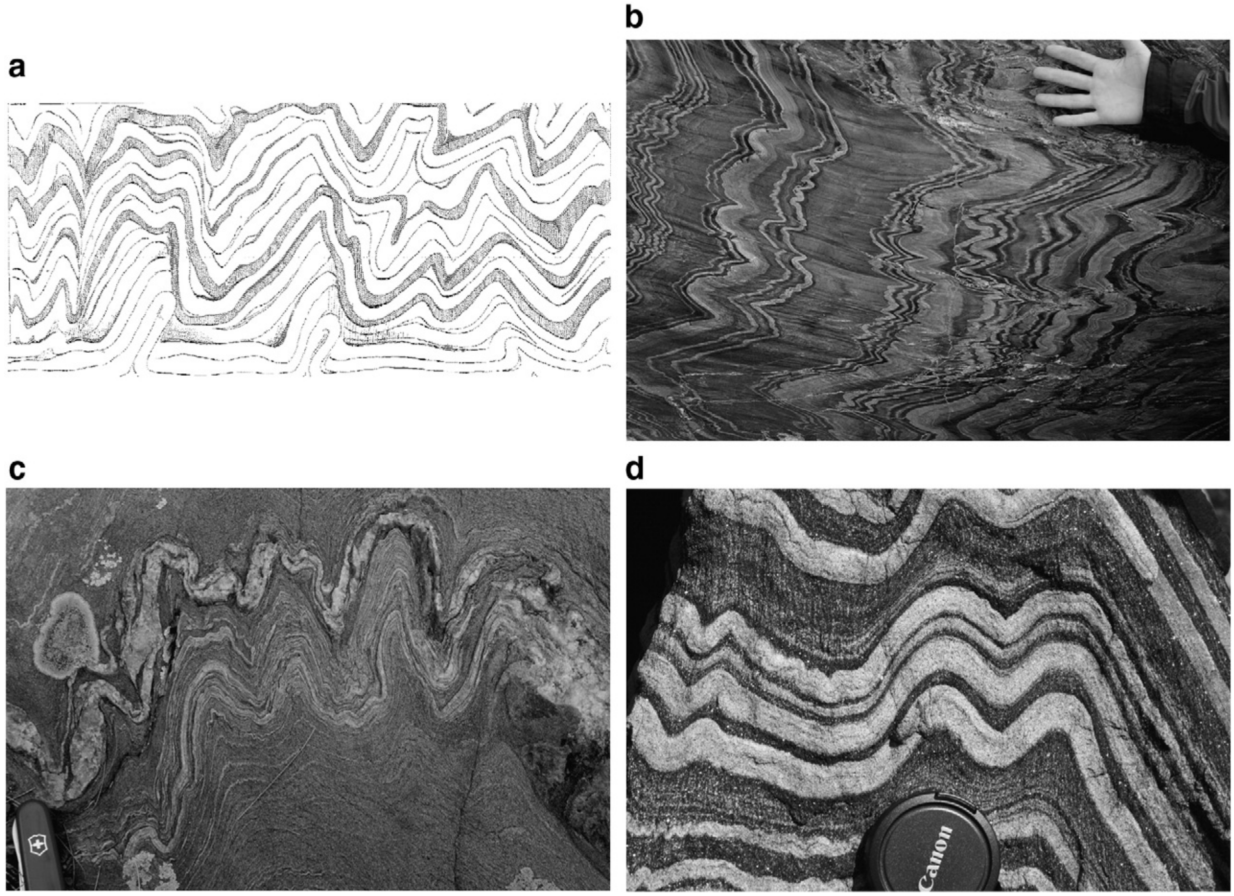


**Fig. 1.** Examples of localised folds. (a) Experimentally produced folds in layers of cloth (Hall, 1815). Model is ca. 1 m across. (b) Folds in fine grained slate, Cornwall. UK. Photo: Tim Dodson. (c) Folds. Cap de Creus. Spain. (d) Folds from Harvey's Retreat, Kangaroo Island, Australia.

nonlinear material or geometrical behaviour of any kind (Johnson and Fletcher, 1994; Mancktelow, 1999, 2001; Schmalholz, 2006; Zhang et al., 1996, 2000). This issue was considered in some depth by Biot (1965) and Biot et al. (1961) and an analytical solution to this problem is given for finite deformations by Muhlhaus et al. (1994, pp 228–231) for Newtonian viscous materials in coaxial deformation histories with constant velocity boundary conditions. That analysis shows that initial perturbations with a wavelength much smaller than the Biot wavelength will grow slowly compared to initial imperfections with wavelengths close to or larger than the Biot wavelength. Individual large wavelength imperfections result in localised fold packets. The issue is: *Are all irregularities in natural fold systems derived from initial imperfections or are there other ways of inducing such irregularity?* The influence of initial perturbations has been the overwhelming emphasis in the geological literature to the exclusion of considerations of softening nonlinear behaviour as it was for many years in the mechanics literature (Augusti, 1964; Budiansky and Hutchinson, 1964; Koiter, 1963; Thompson and Hunt, 1973; Ziegler, 1956). However it is widely recognised in the more recent mechanics literature that other mechanisms of inducing an irregular or localised buckling response involve the development of some form of softening nonlinearity in the geometry or constitutive behaviour of the materials involved and that this process is distinct from the Biot model (Tvergaard and Needleman, 1980; Whiting and Hunt, 1997). In this paper we explore situations where the fold irregularity arises as a natural consequence of the nonlinearity of perfect systems. The word *perfect* is meant to imply that there are no imposed imperfections in the system that result in localisation of deformation. If initial imperfections are present they result in a decrease of the load bearing capacity of the system and may control the site and rate of growth of any localised response. However the presence of imperfections is not a necessary condition for the development of localised folding.

In many systems, although the material behaviour remains linear, geometrical nonlinearities can arise, initially or at finite deformations, that induce unstable behaviour (Cross and Greenside, 2009; Hunt and Hammond, in press; Ortiz and Repetto, 1999) and so one could also ask: *In the absence of suitable initial perturbations does nonlinear behaviour emerge in buckling systems at large deflections even in systems that are linear at small deflections?* We have discussed this in some detail for coaxial deformation histories (Hobbs and Ord, 2012) and although some workers including Muhlhaus (1993), Schmalholz (2006), and Schmalholz and Podladchikov (2000) have reported broadening of the dispersion relationship and bifurcation behaviour at finite deflections of linear viscous materials it appears that no nonlinearity arises that leads to localisation. There is a possibility that amongst the many variables and assumptions inherent in the small deflection, thin layer, coaxial deformation theories there remains an aspect that induces non-sinusoidal behaviour in the absence of initial perturbations once thick layers and/or large deflections are taken into account but to date no such effects have been documented (Hobbs and Ord, 2012). We note however that all such work published to date neglects shear stresses parallel to the folding layer; the work in this paper shows that inclusion of such shear stresses for thick layer theories introduces what are essentially geometrical nonlinearities that lead to localisation in otherwise linear systems.

One fundamental aspect of the Biot theory is that in restricting the discussion to linear materials, the force exerted on the folding layer by the embedding matrix must be a linear function of the deflection of the layer (for elastic materials) or, for viscous materials, a linear