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Parametric and non-parametric statistical approaches to the determination of paleostress from dilatant fractures: Application to an Early Miocene dike swarm in central Japan

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ABSTRACT

Several methods have been proposed for determining paleostress states from orientations of dilatant fractures such as dikes and veins. Recently a stochastic inversion method was invented to objectively estimate the principal stress axes and the stress ratio. Whether a fracture is dilated or not is controlled by the balance of the fluid pressure and the normal stress acting on it. The magnitude of normal stress depends on the fracture orientation, which causes anisotropic orientation distribution of dilatant fractures. The inversion method assumes that the orientation distribution of fractures can be approximated by a Bingham distribution, an exponential probability distribution on the unit sphere, of which symmetric axes are interpreted as the principal stress axes. However, it is unknown if the exponential type of distribution function is suitable or not. Here, we examine the distribution functions and propose two improved methods. One method uses the shifted power-law function as the shape of probability distribution, which is more flexible than the Bingham distribution and is applicable to various shapes of orientation distributions. Furthermore, an index of the driving fluid pressure can be estimated with a confidence interval. The other is a non-parametric (distribution-free) method, which can avoid the a priori assumption on the shape of distribution function without significantly losing accuracy or precision. The new methods were applied to an Early Miocene dike swarm formed during the back-arc opening of the Japan Sea. A normal-faulting stress regime with the minimum principal stress axis trending roughly perpendicular to the arc was obtained from the dikes. A moderately high stress ratio and a high fluid pressure were also estimated.

1. Introduction

Orientations of dilatant fractures such as dikes and mineral veins provide clues to the tectonic paleostress under the influence of crustal fluid. Extension fractures have been thought to be perpendicular to the regional minimum compressive principal stress axis (e.g. Anderson, 1951; Nakamura, 1977). However, natural fractures have variations in their orientations to some extent. As is mentioned below, we can explain some types of such variations without assuming spatiotemporal changes of tectonic stress states, and the variations carry information on the other parameters of stress tensors.

Delaney et al. (1986) formulated the criterion for re-opening of pre-existing fractures as

$$P_\mathrm{f} \geq \sigma_\mathrm{n}, \tag{1}$$

where P_f is the pressure of the fluid which makes dikes or veins, and σ_n is the tectonic normal stress acting on the fracture surface (Fig. 1). This criterion neglects tensile strength of pre-existing fracture. Even in a uniform and constant stress field the magnitude of normal stress acting on a fracture varies with its orientation (Fig. 2). Then the criterion (Eq. (1)) restricts the range of orientations of dilatant fractures.

Jolly and Sanderson (1997) proposed a graphical method to determine stress conditions from the range of fracture orientations (Fig. 3a). Let σ_1, σ_2 and σ_3 be the maximum, intermediate and minimum compressive principal stresses. The feasible range of fracture poles should be centered by σ_3-axis. If the fluid pressure satisfies $P_\mathrm{f}<\sigma_1$, there is a blank region centered by σ_1-axis. After specifying σ_3- and σ_1-axes as the orientations with the maximum and minimum frequencies of fracture poles, σ_2-axis can be determined so as to be perpendicular to both σ_1- and σ_3-axes. The determination of principal axes can be achieved numerically by utilizing eigenvectors of orientation–distribution tensor (Scheidegger, 1965; Woodcock, 1977). The feasible range of fracture poles is not always concentric around σ_3-axis but tends to extend broader toward σ_3-axis than toward σ_1-axis. Jolly and Sanderson (1997) proposed to determine the stress ratio $\Phi = (\sigma_2 - \sigma_3)/(\sigma_1 - \sigma_3)$ from the geometry of feasible range in

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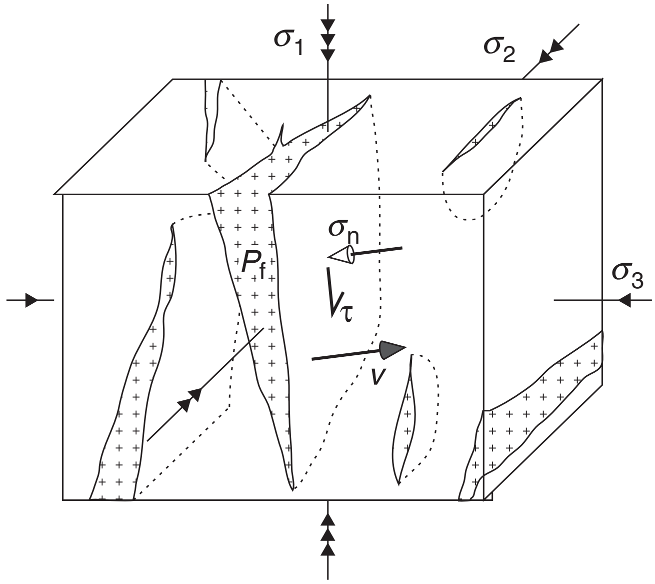

Fig. 1. A schematic drawing of dilatant fractures in a rock mass. In a uniform and constant stress state symbolized by σ_1-, σ_2- and σ_3-axes, a dilatant fracture occurs if the fluid pressure P_f exceeds the tectonic normal stress σ_n. The shear stress τ and normal stress σ_n depend on the orientation of fracture described by the unit normal vector v.

relation to the fluid pressure level (Fig. 3a). Φ ranges from 0 (axial compression) to 1 (axial tension), which represents the shape of stress ellipsoid. Consequently, the purpose of an analysis of dilatant fractures is to constrain the combination of the three principal stress axes and the stress ratio. These independent four variables of a stress tensor are mathematically expressed by a normalized symmetric matrix, which is called 'reduced stress tensor' in the methodology of stress tensor inversion from fault-slip data (e.g. Angelier, 1989). In this study, σ_1 and σ_3 are normalized to be 1 and 0, respectively.

Although the Jolly and Sanderson's method has been applied to natural dilatant fractures (e.g. André et al., 2001; Mazzarini and Isola, 2007), there is a difficulty in the recognition of the border of feasible region on stereograms (Fig. 3a). In many cases the frequency of poles to fractures gradually diminishes toward the border. We have proposed that the problem can be solved by assuming that the variation of frequency reflects the difference of tectonic normal stress arising from the difference of fracture orientation (Fig. 2). Our method fits a Bingham distribution (Bingham, 1974), an exponential probability distribution on sphere, to the orientation distribution of poles to fractures (Fig. 3b). The Bingham stochastic model carries parameters which can be interpreted as those of a reduced stress tensor. The symmetric axes of the optimized Bingham distribution represent the principal stress axes. The anisotropy of distribution on sphere indicates the stress ratio (Fig. 2). Our approach seemed to have succeeded in analyzing epithermal quartz vein swarm in an area in southern Japan and a normal-faulting tectonic stress was obtained.

However, the validity of the exponential stochastic model has not been examined. There possibly is an orientation distribution which is not suitable for the approximation by an exponential function. This study proposes two improved methods. One employs a stochastic model of shifted power-law function which has a larger degrees of freedom than the exponential function. The new function is expected to express various types of decreasing function flexibly (Fig. 3c). The other modified method is non-parametric without any stochastic model. This method searches for a stress state which optimizes the rank correlation coefficient between fracture frequencies and normal stress magnitudes. Its advantage is in the exclusion of a priori assumption on the shape of orientation distribution function. For the purpose of comparing the methods, this paper presents analyses of simulated and natural datasets.

2. Method

Let $\boldsymbol{\sigma}$ and $\boldsymbol{\sigma}_0$ be a stress tensor and its expression in the principal coordinate system, respectively. They can be written as

$$\boldsymbol{\sigma}_0 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \Phi & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{2}$$

and

$$\boldsymbol{\sigma} = \mathbf{Q}^{\mathrm{T}}\boldsymbol{\sigma}_0\mathbf{Q}, \tag{3}$$

where $\mathbf{Q}$ is the orthogonal matrix as the coordinate rotation operator and the superscript T denotes transpose of matrix. When σ_1 and σ_3 are normalized to be 1 and 0, σ_2 corresponds to the stress ratio Φ by definition. Given a unit normal of fracture plane $\vec{v} = (x, y, z)^{\mathrm{T}}$, where x, y and z are the Cartesian coordinates in the physical space, and its expression in the principal coordinate system $\vec{v}_0 = (x_0, y_0, z_0)^{\mathrm{T}}$, the magnitude of normal stress is calculated to be

$$\sigma_{\mathrm{n}}\left(\vec{v}; \boldsymbol{\sigma}\right) = \vec{v}^{\mathrm{T}}\boldsymbol{\sigma}\,\vec{v} = \vec{v}_0^{\mathrm{T}}\boldsymbol{\sigma}_0\vec{v}_0 = x_0^2 + \Phi y_0^2. \tag{4}$$

Our assumption is that the probability of dilatant fracturing to have an orientation $\vec{v}$ is written as a monotonic decreasing function of

$$P\left(\vec{v}; \boldsymbol{\sigma}\right) = f\left(\sigma_{\mathrm{n}}\left(\vec{v}; \boldsymbol{\sigma}\right)\right)\mathrm{d}A, \tag{5}$$

where f is an arbitrary decreasing function and dA is the area element of the unit sphere on which the end point of $\vec{v}$ lies. The nature of stress tensor inversion methods discussed in this paper is the optimization of $\boldsymbol{\sigma}$ in Eq. (5) so as to fit P to the observed frequency distribution of fracture orientations ($\vec{v}$). The formula of f can be freely chosen and three methods are defined and examined below.

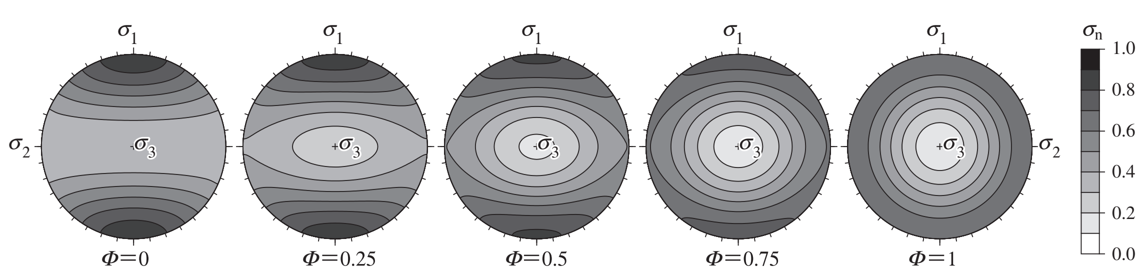

Fig. 2. Orientational distribution of normal stress magnitude for various values of stress ratio Φ. Lower-hemisphere and equal-area projection. Gray scale colors at poles of fractures indicate normalized magnitude of normal stress ($0 \le \sigma_n \le 1$). The principal stress magnitudes are normalized as $\sigma_1 = 1$ and $\sigma_3 = 0$.