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Modelling of the hamstring muscle group by use of fractional derivatives

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ABSTRACT

This paper deals with a viscoelastic model of the hamstring muscle group. The model includes fractional derivatives of stretching force and elongation, as well as restrictions on the coefficients that follow from the Clausius–Duhem inequality. On the basis of a ramp-and-hold type of experiment, four rheological parameters have been calculated by numerical treatment ab initio. Riemann–Liouville fractional derivatives were approximated numerically using the Grünwald–Letnikov definition. Obtained results were verified by use of the Laplace transform method. The stretching force in time domain involves Mittag-Leffler-type function.

1. Introduction

In human anatomy, a *hamstring* refers to one of the tendons that make up the borders of the space behind the knee, but in the modern anatomical context now includes the muscles of the upper leg: the semitendinosus, the semimembranosus, and the biceps femoris. These muscles, together with corresponding tendons, form the hamstring muscle group. The aim of this work is to propose a simplified viscoelastic fractional–derivative model of the hamstring muscle group.

During passive stretch, the muscle-tendon unit is considered to have viscoelastic response [1]. Viscoelastic material, when stretched to a new constant length, the analogous static stretching technique, will decline in tension over time, as described by Magnusson et al. [2]. However, according to Catania and Sorrentino [3], not all models which arise in applications are suitable for describing viscoelastic behavior. When studying dynamics of that kind of material, the selection of an appropriate rheological model is of great importance.

Because of the fact that stress is proportional to the zeroth derivative of strain for solids and to the first derivative of strain for fluids, it is natural to suppose that, for materials that have properties of both solids and fluids (viscoelastic materials), stress may be proportional to the strain derivative of noninteger order α, where $0 < \alpha < 1$, [4]. Namely, fractional calculus based constitutive models are a powerful extension of the standard integer calculus based models, that offer a new alternative for describing biomechanical properties of normal, diseased and healing tissues [5].

In what follows, by use of the generalized Zener model, we intend to propose a new mathematical model of the hamstring muscle group. In doing so, we plan to use the existing experimental data and the methods described in [4], and the Laplace transform, as applied in Petrovic et al. [6]. The important property of the generalized Zener model is that it is able to predict behavior of the viscoelastic material with significant accuracy, including only four parameters, [6]. We expect that four constants included in the model, determined from the stress relaxation experiment, could give useful information on the state of the muscles.

2. Methods

The simplified mechanical model of a hamstring muscle group and human leg is introduced and is similar to the one presented in Tozeren [7]. The hamstring muscle group is modelled by a viscoelastic rod (Fig. 1). Lengths *a*, *b*, *c* and *d* depend

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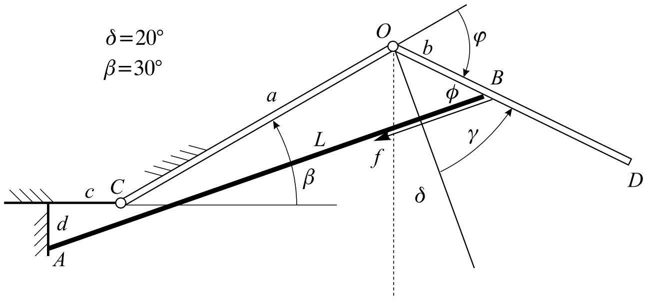

Fig. 1. Diagram of the system under consideration.

on an observed sample, so they are considered as known quantities. During the movement of the lower leg *OD* the length *L* of the muscle depends only on the angular position of the lower leg.

In Fig. 1 *a* denotes the upper leg length, *b* stands for the distance between the knee and the position where the hamstring is tied to the lower leg, while *c* and *d* describe the connection between the hamstring and the pelvis.

The length of the viscoelastic rod representing the muscle can be derived from the following equation

$$L(\gamma) = \frac{b + a\cos(5\pi/9 - \gamma) + c\cos(5\pi/9 - \gamma - \beta) - d\sin(5\pi/9 - \gamma - \beta)}{\cos\phi} \tag{2.1}$$

where

$$\tan\phi = \frac{a\sin(5\pi/9 - \gamma) + c\sin(5\pi/9 - \gamma - \beta) + d\cos(5\pi/9 - \gamma - \beta)}{b + a\cos(5\pi/9 - \gamma) + c\cos(5\pi/9 - \gamma - \beta) - d\sin(5\pi/9 - \gamma - \beta)}. \tag{2.2}$$

Elongation of the rod reads

$$x(\gamma) = L(\gamma) - L(0). \tag{2.3}$$

The correct choice of a rheological model plays an important role in the testing of viscoelastic materials. The model should enable good agreement with experimental data and, at the same time, contain as few parameters as possible. In recent studies, it has been shown that, in case of viscoelastic materials, the generalized Zener model, which comprises fractional derivatives, has advantages over models that include integer order derivatives (Zener or Kelvin-Voight model) [3]. For the generalized Zener model, which we use here, the constitutive equation has the following form:

$$f + \tau_{f\alpha} f^{(\alpha)} = E\left(x + \tau_{x\alpha} x^{(\alpha)}\right), \quad 0 < \alpha < 1, \tag{2.4}$$

where $f^{(\alpha)}$ and $x^{(\alpha)}$ are fractional time derivatives of a force and elongation given in the standard Riemann–Liouville form, as given by Gorenflo and Mainardi in [8]:

$$[g(t)]^{(\alpha)} = \frac{\mathrm{d}^{\alpha}}{\mathrm{d}t^{\alpha}}[g(t)] = \frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{1}{\Gamma(1-\alpha)}\int_0^t \frac{g(\tau)}{(t-\tau)^{\alpha}}\mathrm{d}\tau\right]. \tag{2.5}$$

In Eq. (2.5) Γ stands for Euler Gamma function, $\tau_{f\alpha}$ and $\tau_{x\alpha}$ are constants of dimension $[\text{time}]^{\alpha}$, while the constant $E = E_{\alpha}A/L(0)$ contains the modulus of elasticity E_{α}, the cross-sectional area *A* of the viscoelastic rod and its initial length $L(0)$. Note that there exist fundamental restrictions on the coefficients of the model, that follow from the second law of thermodynamics, [9]

$$E > 0, \quad \tau_{f\alpha} > 0, \quad \tau_{x\alpha} > \tau_{f\alpha}. \tag{2.6}$$

We shall use this model to predict viscoelastic behavior of the hamstring muscle group during stress relaxation. Four constants ($\alpha, E, \tau_{f\alpha}, \tau_{x\alpha}$) describing the model will be determined on the basis of the experimental research on viscoelastic stress relaxation during static stretch of hamstring muscles, as given in [2]. The change of angle γ and elongation *x* of the viscoelastic rod during the experiment is shown in Fig. 2, where a ramp-and-hold type of relaxation experiment can be recognized. Although more realistic than the classical stress relaxation experiment, this type of experiment is not often encountered. During phase 1 of the experiment, the lower leg moves from the initial position defined by the fixed angle δ, at the angular speed of $\dot{\gamma} = 5\,°/\text{s}$ to its final position $\gamma = 80°$, where it stays until the end of the experiment (phase 2), i.e. during the static phase (phase 2) the angle γ remains stationary.

During the experiment, the passive torque *M* [Nm], which equals the moment of the force *f* in the viscoelastic rod for the point *O*, is measured and represented by the following relation

$$M = f\,b\,\sin\phi. \tag{2.7}$$

In order to apply our model to experimental data, seven values of the passive torque were chosen from the stress relaxation curve presented in Fig. 4 of the paper of Magnusson et al. [2]. The appropriate muscle forces $f_i (i = 1, 2, \ldots, 7)$