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Short Communication

Criticality in earthquakes. Good or bad for prediction?

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ABSTRACT

Through the analysis of the correlation functions in simulations of an earthquake model, the critical properties of the system are studied. Simulations are performed in a more realistic modification of the Olami–Feder–Christensen model of earthquakes and result in uncorrelated avalanches distributed following a power-law with weak signs of foreshocks and aftershocks. The spatial autocorrelation function of the system and other structural variables are computed in every step of the simulation. The spatial autocorrelation between points separated from each other by a constant distance equal to 1/4 and 1/8 of the linear size of the system shows large variations, temporally correlated with the time series of avalanche size; i.e., spatial correlation values are in average very high before a large earthquake, very small after a large earthquake and they evolve between these two states. However, the temporal average of the spatial autocorrelation over the whole simulation shows values close to zero, result that is in contradiction with the idea that the correlation length is of the same order as the linear size of the system (diverging in an infinite system), which is the main signature of a critical scenario. By averaging the autocorrelation in smaller time windows, the critical properties of temporal states can be used as an indication of upcoming catastrophic events. The structural variables are also correlated with the occurrence of large avalanches, suggesting the possibility of monitoring these variables in order to achieve prediction.

1. Introduction

With the pioneering work of Wegener (Wegener, 1929) and the establishment of the plate tectonics theory, scientists started to understand the cause and nature of earthquakes. However, statistical studies appeared much earlier (Omori, 1894), urged by the necessity of predicting earthquakes. When, where and how big is the next earthquake have been the questions to answer, and a considerable progress have been achieved concerning the two last ones, i.e., zones with high activity are known and risks can be evaluated (Reiter, 1991), which is essential in building design, planning of land use, estimation of earthquake insurance, etc. However, answering "when" has been much more difficult. Some progress in temporal dependence has also been achieved (Shimazaki and Nakata, 1980; Davis et al., 1989), but attempts to reduce or fix the time window in forecasts, by the analysis of precursors or pattern recognition tools, have generally failed (Geller, 1997; Kagan, 1997a). In 1989 Bak and Tang (1989) classified earthquakes as "Self-organised critical" (SOC) phenomena (Bak et al., 1987). This classification, in addition to the lack of success in predictability, has developed the idea that the crust is at a critical state where a minor perturbation can trigger an earthquake of any size and duration, making them inherently unpredictable (Geller et al., 1997; Main, 1999). This paper will focus on this analogy between critical systems and earthquake behaviour as well as the consequences of criticality for the predictability of the systems. Besides the earthquakes' domain, the results presented here touch other phenomena: snow avalanches (Birkeland and Landry, 2002), solar flares (Hamon et al., 2002), evolution (Sneppen et al., 1995), stock markets (Lee et al., 1998), superconducting vortices (Altshuler and Johansen, 2004; Altshuler et al., 2004), piles of grains (Frette et al., 1996; Altshuler et al., 2001; Aegerter et al., 2003), etc; where the unpredictable character of power-law distributed events often has been taken for granted because of their "critical" properties.

1.1. Critical phenomena

In Physics, the classical scenario of critical phenomena takes place during a second order phase transition (Stanley, 1987). The text-book example is the transition where a permanent magnet loses its magnetism: its magnetic properties cease when the temperature is increased above a certain critical temperature *Tc*. Below this temperature all the spins are aligned in the same direction, creating a magnetic field. Large fluctuations in spins do not occur at low temperatures, so the system will remain unchanged. Above the critical temperature the spins' directions are random and change direction randomly, frequently and individually. The

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system is already disordered, so no large-scale changes will happen. There is no overall magnetic field. However, at the critical temperature itself large fluctuations occur and different snapshots of the system show different patterns — but all of the patterns will be statistically similar, in that clusters of aligned spins are surrounded by areas with spins oriented in the opposite direction. The clusters are of all sizes and their distribution follows a power-law. Three characteristic of the critical state will be used in our analysis:

a) *Divergence of the correlation length (ξ)*: the temporal average of the spatial autocorrelation function

$$\langle C_A s(d,t)\rangle_t = \left\langle \frac{\sum (f(x,y)f(x',y')) - \langle f(x,y)\rangle^2}{\sum (f(x,y) - \langle f(x,y)\rangle)^2} \right\rangle_t \quad (1)$$

where f(*x*,*y*) represents the structure of the system (in two dimensions) and *d* corresponds to the distance between (*x*,*y*) and (*x*′,*y*′), can generally be fitted ($\forall T \neq Tc$) as an exponential decay in the form $C_A s(d) \sim \exp(d/\xi)$. At the critical point, the correlation length ξ is proportional to the linear size of the system (diverging in an infinite system). The temporal average is necessary because the calculus is performed in a snapshot of the dynamics (a microstate) and any physical measure implies an average over many different microstates, which is equivalent to a temporal average if the system is ergodic (Newman and Barkema, 1999).

b) *Divergence of the correlation time (τ)*: the temporal autocorrelation function

$$C_A t(t) = \frac{\sum (f(t_i)f(t_i + t)) - \langle f(t_i)\rangle^2}{\sum (f(t_i) - \langle f(t_i)\rangle)^2} \quad (2)$$

can be fitted as an exponential decay in the form $C_A t(t) \sim \exp(t/\tau)$. Far from the critical point, the correlation time τ is small, so the system will quickly recuperate from a perturbation. At the critical point, τ diverges due to the fact that the system hesitates between the two states and the dynamics turn slow; perturbations can move the system away from its equilibrium state during long periods of time. Both ξ and τ present power-law dependences with the reduced temperature $t = (T - Tc)/Tc$ in the way $\xi = |t|^{-\nu}$ and $\tau = |t|^{-z\nu}$, so they relate to each other through $\tau = \xi^z$ (Newman and Barkema, 1999).

c) As the size of the system increases, the transition between the two states becomes sharper, and it is infinitely sharp in an infinite system (Newman and Barkema, 1999).

2. The model

Simulation in a very simple cellular automaton model of earthquakes will guide us in the discussion of criticality and predictability aiming towards a more general frame. By mapping the Burridge–Knopoff spring-block model (1967) into a cellular automaton, Olami, Feder and Christensen (OFC) demonstrated that through introducing dissipation in the system, the exponent of the power-law of the avalanche size distribution can be tuned (Olami et al., 1992). If 20% of the energy is lost in every redistribution of forces after one block slips, the distribution resembles the Gutenberg–Richter law (Christensen and Olami, 1992). The simulations presented in this article take place on a lattice of $L \times L$ sites ($L = 128, 256, 512$) with open boundary conditions, where a few modifications to the original OFC model make the system more realistic (Ramos et al., 2006). The spring-block model consists of a two dimensional array of blocks on a flat surface. Each block is connected by means of springs with its four nearest neighbours, and in the vertical direction, to a driving plate which moves horizontally at velocity *v*. When the force acting on a block overcomes the static friction of the surface, the block slips. A redistribution of forces then takes place in the neighbours that eventually triggers new displacements. In our model, the force on each block is stored in a site of the lattice, and the static friction thresholds are distributed randomly following a Gaussian centred at 1.0 with a standard deviation equal to 0.001. Starting from zero, a constant amount of force equal to 10^{-4} is added to every site in each step. Eventually, one or several sites can reach their thresholds; the site is then set to zero and a fraction α of its force is redistributed to its neighbours. If one of the neighbours reaches its threshold, the process is repeated until all the sites have their values below the threshold. After a site is set to zero, a new random threshold is imposed (a new value of static friction coefficient for a new place). The avalanche size is defined as the number of sites involved in the avalanche before the force is raised again. The values of α decrease with the dissipation (percentage of lost energy, E_L) according to $\alpha = (1 - E_L/100)/4$ and they have been chosen randomly following a Gaussian distribution centred at 0.2 with a standard deviation σ_α. When a block slips α is changed.

The rules are the following:

At $t = 0$: $F(x,y) = 0$; threshold(x,y) = Gaussian(mean = 1, sd = 0.001) $\forall x,y \in [0,L)$
$\forall t > 0$: $F(x,y) = F(x,y) + 0.0001$ $\forall x,y \in [0,L)$
  While $F(x,y) \geq$ threshold(x,y)
    {alpha = Gaussian(mean = 0.2, sd = 0.005)
    $F(x+1,y) = F(x+1,y) + F(x,y)^*$ alpha
    $F(x-1,y) = F(x-1,y) + F(x,y)^*$ alpha
    $F(x,y+1) = F(x,y+1) + F(x,y)^*$ alpha
    $F(x,y-1) = F(x,y-1) + F(x,y)^*$ alpha
    $F(x,y) = 0$
    Threshold(x,y) = Gaussian(mean = 1, sd = 0.001)}
  Avalanche size = number of sites involve in the avalanche.

These simulations continue the studies developed by (Ramos et al., 2006) whose main focus corresponded to the situation where the dissipation is uniform and constant ($\sigma_\alpha = 0$). As a result, a nontrivial quasi-periodicity rules the dynamics, suggesting that the earthquake's natural behaviour is a quasi-periodic state, and that the variations or absence of periodicity is due to changes in the dissipative regime and/or in the relative velocity of the plates and/or in the amount of energy that can be stored in a given zone between two tectonic plates (Ramos et al., 2006). See also (Kagan, 1997b) for a discussion about quasi-periodicity in earthquakes. The OFC model also displays quasi-periodic behaviour (Ramos et al., 2006), but is less strong that in our model, hidden in some way in the more efficient manner to load the system. All the other known features of the OFC model remain (coexisting with the quasi-periodicity), and the distributions of avalanches display power-law behaviours with exponents that decrease with the dissipation. For $\alpha = 0.2$ (20% of the energy is dissipated) the exponent is equal to −1.91. With the aim of removing the periodicity, in order to reach a more realistic scenario, the values of the dissipation simulated in this article are distributed following a Gaussian. For $\sigma_\alpha = 0.005$ the distribution of avalanches suffers no changes. However, Fig. 1b shows that there is almost no correlation between the avalanches; just small bumps slightly above the noise level in the autocorrelation function of large avalanches, indicating the loss of quasi-periodicity. For $\sigma_\alpha = 0.01$ small changes start to be noticed in the avalanche distribution and for $\sigma_\alpha = 0.02$ the distribution in no longer a power-law. That is the reason why the simulations have been performed with $\alpha = 0.2$ and $\sigma_\alpha = 0.005$: to get uncorrelated avalanches distributed according to a power-law with an exponent that resembles the Gutenberg–Richter law (Gutenberg and Richter, 1956).

Following the work introduced by Ramos et al. (2009), where for the first time experimental SOC avalanches have been predicted, global structural variables have been measured in every step of the simulations. They are the average force, the standard deviation of the average force and the spatial correlation between all pairs of sites separated from each other 1/4 and 1/8 of the linear size of the system *L*.