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Bounded variation double sequence space of fuzzy real numbers

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ABSTRACT

In this paper we have introduced the notion of fuzzy real-valued bounded variation double sequence space ${}_2bv_F$. We have studied some of its properties like convergence free, solidness, symmetricity, monotonicity, etc. We have proved some inclusion results too.

1. Introduction

The concept of fuzzy set was introduced by Zadeh [1]. Based on this, sequences of fuzzy numbers have been introduced by different authors and many important properties have been investigated. Applying the notion of fuzzy real numbers, fuzzy real-valued sequence were introduced and investigated by many researchers on sequence space.

A fuzzy real number X is a fuzzy set on R i.e. a mapping $X : R \to I(=[0, 1])$, associating each real number t, with its grade of membership $X(t)$.

The α-level set of a fuzzy real number X is denoted by $[X]_\alpha$, $0 < \alpha \le 1$, where $[X]_\alpha = \{t \in R : X(t) \ge \alpha\}$.

A fuzzy real number X is said to be *upper semi-continuous* if for each $\varepsilon > 0$, $X^{-1}([0, a + \varepsilon))$, for all $a \in I$ is open in the usual topology of R.

If there exists $t \in R$ such that $X(t) = 1$, then the fuzzy real number X is called *normal*.

A fuzzy real number X is said to be *convex*, if $X(t) \ge X(s) \wedge X(r) = \min(X(s), X(r))$, where $s < t < r$.

The class of all *upper semi-continuous*, *normal* and *convex* fuzzy real numbers is denoted by $R(I)$.

Let $X, Y \in R(I)$ and the α-level sets be $[X]_\alpha = [a_1^\alpha, a_2^\alpha]$, $[Y]_\alpha = [b_1^\alpha, b_2^\alpha]$, $\alpha \in [0, 1]$.

The absolute value of $X \in R(I)$ is defined by (One may refer to Kaleva and Seikkala [2].)

$$|X|(t) = \begin{cases} \max\{X(t), X(-t)\}, & \text{for } t \ge 0 \\ \bar{0}, & \text{otherwise.} \end{cases}$$

The additive identity and multiplicative identity of $R(I)$ are denoted by $\bar{0}$ and $\bar{1}$ respectively.

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Let D be the set of all closed bounded intervals $X = [X^L, X^R]$. Then we write

$$X \leq Y \quad \text{if and only if} \quad X^L \leq Y^L \quad \text{and } X^R \leq Y^R$$

and

$$d(X, Y) = \max\{|X^L - Y^L|, |X^R - Y^R|\}, \quad \text{where } X = [X^L, X^R] \text{ and } Y = [Y^L, Y^R].$$

Then clearly (D, d) is a complete metric space.

Now define $\bar{d} : R(I) \times R(I) \to R$ by

$$\bar{d}(X, Y) = \sup_{0 \leq \alpha \leq 1} d(X^\alpha, Y^\alpha), \quad \text{for } X, Y \in R(I).$$

2. Definitions and preliminaries

Throughout w, c, c_0, ℓ_∞, and ℓ_1 denote the classes of *all*, *convergent*, *null*, *bounded* and *absolutely summable* sequences of real or complex terms respectively.

A fuzzy real-valued double sequence is a double infinite array of fuzzy real numbers. We denote a fuzzy real-valued double sequence by $\langle X_{nk} \rangle$, where X_{nk} are fuzzy real numbers for each $n, k \in N$.

The initial works on double sequences of real or complex terms is found in Bromwich [3]. Hardy [4] introduced the notion of regular convergence for double sequences of real or complex terms. The works on double sequence was further investigated by Moričz [5], Basarir and Solancan [6], Tripathy and Sarma [7] and many others.

We procure the following definitions on fuzzy real-valued double sequences:

Definition. A fuzzy real-valued double sequence $\langle X_{nk} \rangle$ is said to be *convergent in Pringsheim's sense* to the fuzzy real number X, if for every $\varepsilon > 0$, there exists $n_1 = n_1(\varepsilon)$, $k_1 = k_1(\varepsilon)$, such that $\bar{d}(X_{nk}, X) < \varepsilon$ for all $n \geq n_1$ and $k \geq k_1$.

Definition. A fuzzy real-valued double sequence $\langle X_{nk} \rangle$ is said to be *bounded* if

$$\sup_{n,k} \bar{d}(X_{nk}, \bar{0}) < \infty,$$

equivalently, if there exists $\mu \in R(I)^*$, such that $|X_{nk}| \leq \mu$ for all $n, k \in N$,
Where $R(I)^*$ denotes the set of all positive fuzzy real numbers.

Definition. A fuzzy real-valued double sequence $\langle X_{nk} \rangle$ is said to be *regularly convergent* if it is convergent in Pringsheim's sense and the followings hold:

For a given $\varepsilon > 0$, there exists $n_0 = n_0(\varepsilon, k)$ and $k_0 = k_0(\varepsilon, n)$ such that

$$\bar{d}(X_{nk}, L_k) < \varepsilon, \text{ for all } n \geq n_0, \text{ for some } L_k \in R(I) \text{ for each } k \in N, \text{ and}$$
$$\bar{d}(X_{nk}, M_n) < \varepsilon, \text{ for all } k \geq k_0, \text{ for some } M_n \in R(I) \text{ for each } n \in N.$$

Definition. A fuzzy real-valued double sequence space E_F is said to be *normal* (*or solid*) if $\langle Y_{nk} \rangle \in E_F$, whenever $|Y_{nk}| \leq |X_{nk}|$ for all $n, k \in N$ and $\langle X_{nk} \rangle \in E_F$.

Definition. A fuzzy real-valued double sequence space E_F is said to be *monotone* if E_F contains the canonical pre-images of all its step spaces.

Definition. A fuzzy real-valued double sequence space E_F is said to be *symmetric* if $\langle X_{\pi(n),\pi(k)} \rangle \in E_F$, whenever $\langle X_{nk} \rangle \in E_F$, where π is a permutation of N.

Definition. A fuzzy real-valued double sequence space E_F is said to be *convergence free* if $\langle X_{nk} \rangle \in E_F$ whenever $\langle Y_{nk} \rangle \in E_F$ and $Y_{nk} = \bar{0}$ implies $X_{nk} = \bar{0}$.

Hardy [4] introduced the notion of regular convergence for double sequences.

A double sequence $\langle a_{nk} \rangle$ is said to converge *regularly* if it converges in *Pringsheim's sense* and in addition the following limits hold:

$$\lim_{k \to \infty} a_{nk} = L_n \ (n \in N) \text{ exist},$$
$$\lim_{n \to \infty} a_{nk} = J_k \ (k \in N) \text{ exist}.$$

The notion of difference sequence space for complex terms was introduced by Kizmaz [8], defined by

$$Z(\Delta) = \{(x_k) : (\Delta x_k) \in Z\}, \quad \text{for } Z = \ell_\infty, c, c_0, \text{ where } \Delta x_k = x_k - x_{k+1}, \text{ for all } k \in N.$$

The notion was further investigated by Tripathy [9,10], Tripathy and Mahanta [11] and many others.