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Short communication

Long-range connective sandpile models and its implication to seismicity evolution

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ABSTRACT

We propose a new variant of the sandpile model, the long-range connective sandpile model, by means of introducing randomly internal connections between two separated distant cells. The long-range connective sandpile model demonstrates various self-organized critical states with different scaling exponents in the power-law frequency-size distributions. We found that a sandpile with higher degree of randomly internal long-range connections is characterized by a higher value of the scaling exponent for the distribution, whereas the nearest neighbor sandpile is possessed of a lower scaling exponent. Our numerical experiments on the long-range connective sandpile models imply that higher degree of random long-range connections makes the earthquake fault system more relaxant that releases accumulated energy more easily and produces fewer catastrophic events, whereas lower degree of long-range connections possibly caused by fracture healing very likely motivates accelerating seismicity of moderate events.

1. Introduction

Many geophysical phenomena are scale invariant and exhibit the power-law distribution (Turcotte, 1997; Dodds and Rothman, 2000), which is the only statistical distribution not including a characteristic scale. A striking example is the Gutenberg–Richter relation for the frequency–magnitude statistics of earthquakes. The scaling exponent and its associated variation is then a matter of fundamental importance in such power-law distribution. Specifically, in the study of seismicity evolution, the scaling exponent in the Gutenberg–Richter relation, which is well known as the *b*-value, has been very often discussed in the literature and considered as a monitoring index related to the forthcoming large earthquakes (Smith, 1986; Urbancic et al., 1992; Wiemer and Wyss, 1994; Henderson et al., 1994; Guo and Ogata, 1995; Legrand et al., 1996; Wyss, 1997; Lapenna et al., 1998; Henderson et al., 1999; Barton et al., 1999; Oncel and Wilson, 2004; Wyss et al., 2004; Mandal et al., 2005; Wu and Chiao, 2006). The reductions in the *b*-value before a large earthquake have been reported in many researches. The reduced *b*-value is probably caused by the quiescence of smaller earthquakes and/or the activation of moderate earthquakes (e.g. Chen, 2003; Chen et al., 2005; Wu and Chiao, 2006). For example, observed before the 1999 M_w 7.6 Chi-Chi, Taiwan earthquake were the quiescence of earthquakes with magnitudes smaller than 4 (Fig. 4 in Wu and Chiao, 2006) and activation of events with magnitudes larger than 5 (Fig. 3 in Chen, 2003).

Numerical experiments in tending to comprehend seismicity had mainly been based on simple conceptual models such as the spring–slider model of Burridge and Knopoff (1967), the sandpile model of Bak et al. (1987), the block structure model of Gabrielov et al. (1990), and the lattice-solid model of Mora and Place (1994). Among them two types of simple cellular automata models are the spring–slider model (Burridge and Knopoff, 1967) and the sandpile model (Bak et al., 1987). In the sandpile model a hallmarked state, which is very well known as the self-organized criticality (SOC) state and characterized by the frequency-size power-law distribution, is established solely because of the dynamical interactions among individual elements of the system. Since the concept of self-organized criticality was introduced in Bak et al. (1987), earthquakes have been identified as an example of this phenomenon in nature (Bak and Tang, 1989; Sornette and Sornette, 1989; Ito and Matsuzaki, 1990) and the observation of the Gutenberg–Richter law has been suggested to be the manifestation of the self-organized critical state of the dynamics of the earthquake faults.

For earthquake studies, the sandpile model sheds new insights into the earthquake physics in addition to those derived from earlier, much complicated spring–slider models (Burridge and Knopoff, 1967; Rundle and Jackson, 1977; Carlson et al., 1994). Here we propose to

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invoke an alternative variant of the sandpile model to demonstrate variations in the scaling exponent of the power-law distribution by means of introducing randomly internal connections between separated distant cells. Note that the internal networks of either the conventional spring–slider or sandpile models are essentially the nearest neighboring networks. We examine the systematic impacts on the scaling exponents of these randomly distant connections in our modified sandpile model. The implications on the earthquake fault systems are then discussed in the end of this paper.

2. Long-range connective sandpile models and its frequency-size distributions

We build our sandpile model by a very simple set of rules that is very similar to the original one (Bak et al., 1987). For a square grid of L by L cells, we randomly throw sands, one at a time, onto the grid. In the original sandpile model, once the total amount of the accumulated sands within a single cell reaches the threshold amount of 4, they will be redistributed to the four adjacent cells (the nearest neighbors) or lost off the edge of the grid. All the cells receiving grains from their neighbors will be checked, and the redistribution would be continued further away if any one of them reaches consequently over the threshold too. For each throw of new grains, the redistribution proceeds until none of the meshes receiving new grains exceeds the threshold. The total amount of cells involved in the redistribution process initiated by a single throw is defined to be the size of the event. Note that the total amount of grains retained within the grid increases linearly in the beginning transient thousands of iterations and then stays at a quasi-static value with small fluctuations.

Our modified rule of randomly internal connections is very similar to the implementation of Watts and Strogatz (1998). For any particular cell, when the accumulated grains exceed the threshold and redistribution occurs, one of the original nearest neighbor connections confronts a chance with prescribed *connective* probability P_c, $0<P_c<1$, of redirecting to a randomly chosen, distant cell and so the original connection is replaced by a randomly chosen mesh that might be faraway from the toppling cell. We may call such version of sandpile model the *long-range connective sandpile* (LRCS) model. Apparently, when $P_c=0$, the LRCS model reduced to the original nearest neighbor sandpile model.

We perform a series of simulations each with 10^6 throws of single grain on a square grid with 50×50 meshes, starting with the original nearest neighbor sandpile ($P_c=0$) and then gradually increasing the connective probability P_c. For each P_c, the frequency-size power law and the manifested SOC state always emerge robustly. The power-law frequency-size distributions demonstrating the scale invariance on the log–log plot for two experiments with $P_c=0$ and $P_c=1$ are very distinct (Fig. 1). The breakdown of the power law at the extreme of great events is obviously due to the limited size of the experimental grid, the so-called finite-size effect.

The frequency-size power law is not a linear-normal relation. Hence, all the data points for the distribution need not to have the same weight as supposed when a linear relation is regressed with a standard least-square method. For example, if the distribution has two large events instead of one, it will change very much the result from using a least-square method. To address the issue of different weighting in a power-law distribution, we adopt a weighted least-square method to assign different fitting weights for different data points of the distribution (Shi and Bolt, 1982). The results lead to a scaling exponent of 1.06 characterizing the frequency-size statistics for the original nearest neighbor sandpile model (Bak et al., 1987), whereas a higher scaling exponent of 1.32 for the LRCS model with $P_c=1$ (Fig. 1).

As gradually increasing the connective probability P_c from 0 to 1, we have found that there is a systematic steady increase of the scaling exponent along with a notable decrease of the amount of total grains staying on the grid (Fig. 2). The comparison suggests that, with higher degree of random long-range connections that is obtained by invoking higher connective probability P_c, the critical state is characterized by a sandpile with more small avalanches involving only a few cells and, consequently, the higher value of the scaling exponent for the frequency-size distribution. In other words, higher degree of random long-range connections actually makes the system more relaxant that releases the potential of building up catastrophic avalanches more easily and produces fewer catastrophic avalanches.

3. Conclusion and discussion

There have been reported that an observed feature associated with the earthquake statistics is a significant increase of moderate earthquakes and a noticeable reduction of the b-value of the Gutenberg–Richter relationship prior to great earthquakes (e.g. Sykes and Jaume, 1990; Henderson et al., 1994; Bowman et al., 1998; Jaume and Sykes, 1999; Chen, 2003; Wu and Chiao, 2006). An immediate implication of our experiments to the temporal evolution of the seismicity is that, as the tectonic loading is slowly driving the earthquake fault system through the cycle of earthquakes leading to the catastrophic event, there might be different phases all bearing characteristics of SOC. The initial phase is characterized by a small but notable random long-range connection with $1>P_c\gg 0$. It then evolves gradually towards more and more regular nearest neighbor connection with $P_c\sim 0$ such that the manifestation is the noted accelerating seismicity of moderate size and a reduction of the b-value (Fig. 2). After the occurrence of some great earthquakes, possibly due to the activity of faults, the change of pore water pressure or the dynamical triggering of seismic waves, the earthquake fault system is very likely to bear with a significant degree of long-range connections with higher P_c. Bursting aftershock activity might well be the manifestation of a raised long-range connection. Then, the system heals gradually along with the tectonic loading and gets back to the initial phase preparing for the next cycle of extreme events.

The mechanism depicted above is utterly different from the temporal growth of *long-range correlation length* on the system in the critical point theory of earthquakes (Bowman et al., 1998; Jaume and Sykes, 1999; Rundle et al., 1999; Rundle et al., 2000). To the critical point theory of earthquakes, the correlation length of the regional stress field grows prior to the catastrophic event. The origin of accelerating seismicity of moderate events lies in the growing

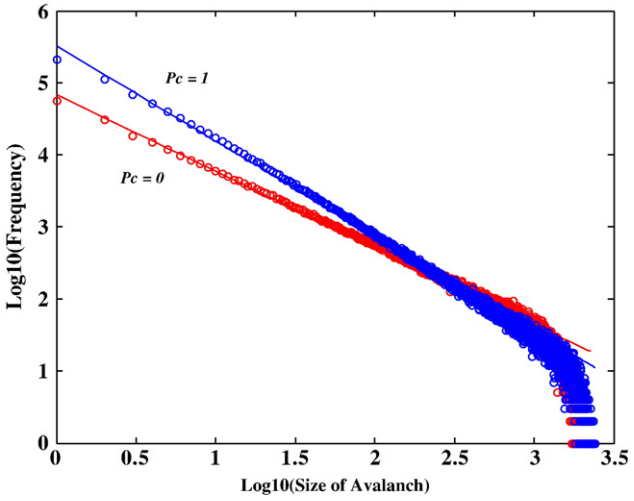

Fig. 1. Power-law frequency-size distributions for two simulations of the long-range connective sandpile models with $P_c=0$ and $P_c=1$, respectively. Also shown are two regression lines, with slopes of 1.06 and 1.32, best fitting the power-law distributions from the weighted least-square method. Note that the long-range connective sandpile with $P_c=0$ is identical to the original nearest neighbor sandpile by Bak et al. (1987).