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On certain subclasses of meromorphic functions associated with certain integral operators

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ABSTRACT

Let Σ denote the class of functions of the form $f(z)=\frac{1}{z}+\sum_{k=1}^{\infty}a_kz^k$ which are analytic in $0<|z|<1$. Two new integral operators P_β^α and Q_β^α defined on Σ are introduced. This paper gives some subordination and convolution properties of certain subclasses of meromorphic functions which are defined by the previously-mentioned integral operators.

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1. Introduction

Let Σ denote the class of functions of the form

$$f(z)=\frac{1}{z}+\sum_{k=1}^{\infty}a_kz^k \tag{1.1}$$

which are analytic in the punctured unit disk $U^*=\{z:0<|z|<1\}=U\setminus\{0\}$, with a simple pole at the origin.

If $f(z)$ and $g(z)$ are analytic in U, we say that $f(z)$ is subordinate to $g(z)$, written $f\prec g$ or $f(z)\prec g(z)(z\in U)$, if there exists a Schwarz function $w(z)$ in U with $w(0)=0$ and $|w(z)|<1(z\in U)$, such that $f(z)=g(w(z)),\ (z\in U)$. If $g(z)$ is univalent in U, then the following equivalence relationship holds true:

$$f(z)\prec g(z)\quad (z\in U)\Leftrightarrow f(0)=g(0)\quad \text{and}\quad f(U)\subset g(U).$$

For functions $f(z)\in\Sigma$, given by (1.1) and $g(z)\in\Sigma$ defined by

$$g(z)=\frac{1}{z}+\sum_{k=1}^{\infty}b_kz^k, \tag{1.2}$$

the Hadamard product (or convolution) of $f(z)$ and $g(z)$ is given by

$$(f*g)(z):=\frac{1}{z}+\sum_{k=1}^{\infty}a_kb_kz^k=:(g*f)(z). \tag{1.3}$$

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Analogous to the operators defined by Jung, Kim and Srivastava [2] on the normalized analytic functions, we now define the following integral operators $P_\beta^\alpha, Q_\beta^\alpha : \Sigma \to \Sigma$:

$$P_\beta^\alpha = P_\beta^\alpha f(z) = \frac{\beta^\alpha}{\Gamma(\alpha)} \frac{1}{z^{\beta+1}} \int_0^z t^\beta \left(\log \frac{z}{t}\right)^{\alpha-1} f(t)\mathrm{d}t \quad (\alpha > 0, \beta > 0; z \in U^*), \tag{1.4}$$

$$Q_\beta^\alpha = Q_\beta^\alpha f(z) = \frac{\Gamma(\beta+\alpha)}{\Gamma(\beta)\Gamma(\alpha)} \frac{1}{z^{\beta+1}} \int_0^z t^\beta \left(1 - \frac{t}{z}\right)^{\alpha-1} f(t)\mathrm{d}t \quad (\alpha, \beta > 0; z \in U^*) \tag{1.5}$$

and

$$J_\beta = J_\beta f(z) = \frac{\beta}{z^{\beta+1}} \int_0^z t^\beta f(t)\mathrm{d}t \quad (\beta > 0; z \in U^*) \tag{1.6}$$

where $\Gamma(\alpha)$ is the familiar Gamma function.

Using the integral representation of the Gamma and Beta functions, it can be shown that

Remark 1. For $f(z) \in \Sigma$ given by (1.1), we have

$$P_\beta^\alpha f(z) = \frac{1}{z} + \sum_{k=1}^{\infty} \left(\frac{\beta}{k+\beta+1}\right)^\alpha a_k z^k, \quad (\alpha > 0, \beta > 0) \tag{1.7}$$

$$Q_\beta^\alpha f(z) = \frac{1}{z} + \frac{\Gamma(\beta+\alpha)}{\Gamma(\beta)} \sum_{k=1}^{\infty} \frac{\Gamma(k+\beta+1)}{\Gamma(k+\beta+\alpha+1)} a_k z^k \quad (\alpha > 0, \beta > 0) \tag{1.8}$$

and

$$J_\beta f(z) = \frac{1}{z} + \sum_{k=1}^{\infty} \frac{\beta}{k+\beta+1} a_k z^k \quad (\beta > 0). \tag{1.9}$$

By virtue of (1.7), (1.8) and (1.9) we see that

$$J_\beta f(z) = P_\beta^1 f(z) = Q_\beta^1 f(z),$$

$$z\left(P_\beta^\alpha f(z)\right)' = \beta P_\beta^{\alpha-1} f(z) - (\beta+1) P_\beta^\alpha f(z) \quad (\alpha > 1, \beta > 0) \tag{1.10}$$

and

$$z\left(Q_\beta^\alpha f(z)\right)' = (\beta+\alpha-1) Q_\beta^{\alpha-1} f(z) - (\beta+\alpha) Q_\beta^\alpha f(z) \quad (\alpha > 1, \beta > 0). \tag{1.11}$$

Now we introduce the following subclasses of Σ associated with the integral operators $P_\beta^\alpha f(z)$ and $Q_\beta^\alpha f(z)$.

Definition 1. For fixed parameters $A, B(-1 \leq B < A \leq 1)$, a function $f(z) \in \Sigma$ is said to be in the class $\Sigma_{\beta,\alpha}^P(\lambda, A, B)$ if

$$-z^2 \left\{(1-\lambda)\left(P_\beta^\alpha f(z)\right)' + \lambda\left(P_\beta^{\alpha-1} f(z)\right)'\right\} \prec \frac{1+Az}{1+Bz} \quad (z \in U), \tag{1.12}$$

where $\alpha > 1$, $\beta > 0$ and $\lambda \geq 0$.

Definition 2. For fixed parameters $A, B(-1 \leq B < A \leq 1)$, a function $f(z) \in \Sigma$ is said to be in the class $\Sigma_{\beta,\alpha}^Q(\lambda, A, B)$ if

$$-z^2 \left\{(1-\lambda)\left(Q_\beta^\alpha f(z)\right)' + \lambda\left(Q_\beta^{\alpha-1} f(z)\right)'\right\} \prec \frac{1+Az}{1+Bz} \quad (z \in U), \tag{1.13}$$

where $\alpha > 1$, $\beta > 0$ and $\lambda \geq 0$.

In this paper, we drive some subordination results of the classes $\Sigma_{\beta,\alpha}^P(\lambda, A, B)$ and $\Sigma_{\beta,\alpha}^Q(\lambda, A, B)$, and investigate several convolution properties of functions which have been defined here by means of the integral operators $P_\beta^\alpha f(z)$ and $Q_\beta^\alpha f(z)$.

2. Preliminaries

To prove our main results, we need the following lemmas.

Lemma 1 ([1], see also [4]). *Let $\phi(z)$ be analytic in U and $h(z)$ be analytic and convex (univalent) in U with $h(0) = \phi(0) = 1$. If*

$$\phi(z) + \frac{z\phi'(z)}{\gamma} \prec h(z)(\mathrm{Re}(\gamma) \geq 0; \quad \gamma \neq 0; \ z \in U), \tag{2.1}$$

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