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Erratum

Erratum to: "Conversion and evaluation for two types of parametric surface constructed by NTP bases" [Comput. Math. Appl. 49(2) (2005) 321–329]

Kan Itsariyawanich, Natasha Dejdumrong *

Department of Computer Engineering, Faculty of Engineering, King Mongkut's University of Technology Thonburi, Bangkok, 10140, Thailand

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ABSTRACT

It is pointed out by Jiang and Wang (2005) [1], that the conversion formula from Bernstein into DP bases is incorrect for all even degrees and the conversion formula from DP into Bernstein bases is incorrect for every degree. Thus, in this paper we give some notes, corrections and new proofs for the relationship between these two NTP bases.

1. Introduction

In paper [1], Jiang and Wang proposed the relationships between Bézier and DP curves in terms of the conversions from Bernstein into DP bases and vice versa. However, it is found out that the conversion formula from Bernstein basis into DP basis is incorrect for even cases (Table 1). Similarly, the conversion formula from DP into Bernstein bases is also incorrect for both odd and even degrees (Table 2). Moreover, their proofs provided in that paper, have contained some mistakes. Thus, in this paper, the polar form approach is used for providing the correct formula and proof for the relationship between Bézier and DP control points.

Consequently, the formulae for the transformation from Bernstein basis into DP basis and the conversion from DP into Bernstein bases are derived and proven, respectively.

2. The relationships between Bézier and DP curves

In this section, the polar form approach is used to prove the relationships between Bézier and DP curves. Both Bézier and DP curves can be defined in terms of their control points and their blending functions. Their basis functions, $\left\{B_i^n(t)\right\}_{i=0}^n$ and $\left\{D_i^n(t)\right\}_{i=0}^n$, respectively, are the polynomials of degree n when $t \in [0, 1]$, consisting of coefficients for each polynomial. Such relationships can be written in the following theorem.

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* Corresponding author.
E-mail address: natasha@cpe.kmutt.ac.th (N. Dejdumrong).

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Table 1
Comparisons of the conversions from DP into Bézier control points.

Degree	Method by Jiang and Wang	Result	Using the polar form approach
$n = 3$	$\mathbf{b}_0 = \mathbf{p}_0$		$\mathbf{b}_0 = \mathbf{p}_0$
	$\mathbf{b}_1 = \frac{2}{3}\mathbf{p}_1 + \frac{1}{3}\mathbf{p}_2$		$\mathbf{b}_1 = \frac{2}{3}\mathbf{p}_1 + \frac{1}{3}\mathbf{p}_2$
	$\mathbf{b}_2 = \frac{1}{3}\mathbf{p}_1 + \frac{2}{3}\mathbf{p}_2$		$\mathbf{b}_2 = \frac{1}{3}\mathbf{p}_1 + \frac{2}{3}\mathbf{p}_2$
	$\mathbf{b}_3 = \mathbf{p}_3$		$\mathbf{b}_3 = \mathbf{p}_3$
$n = 4$	$\mathbf{b}_0 = \mathbf{p}_0$		$\mathbf{b}_0 = \mathbf{p}_0$
	$\mathbf{b}_1 = \frac{1}{4}\mathbf{p}_1 + \frac{3}{4}\mathbf{p}_2$		$\mathbf{b}_1 = \frac{1}{4}\mathbf{p}_1 + \frac{3}{4}\mathbf{p}_2$
	$\mathbf{b}_2 = \frac{3}{2}\mathbf{p}_2$	$\times$	$\mathbf{b}_2 = \mathbf{p}_2$
	$\mathbf{b}_3 = \frac{3}{2}\mathbf{p}_2 + \frac{1}{4}\mathbf{p}_3$		$\mathbf{b}_3 = \frac{3}{4}\mathbf{p}_2 + \frac{1}{4}\mathbf{p}_3$
	$\mathbf{b}_4 = \mathbf{p}_4$		$\mathbf{b}_4 = \mathbf{p}_4$
$n = 5$	$\mathbf{b}_0 = \mathbf{p}_0$		$\mathbf{b}_0 = \mathbf{p}_0$
	$\mathbf{b}_1 = \frac{1}{5}\mathbf{p}_1 + \frac{1}{2}\mathbf{p}_2 + \frac{3}{10}\mathbf{p}_3$		$\mathbf{b}_1 = \frac{1}{5}\mathbf{p}_1 + \frac{1}{2}\mathbf{p}_2 + \frac{3}{10}\mathbf{p}_3$
	$\mathbf{b}_2 = \frac{11}{20}\mathbf{p}_2 + \frac{9}{20}\mathbf{p}_3$		$\mathbf{b}_2 = \frac{11}{20}\mathbf{p}_2 + \frac{9}{20}\mathbf{p}_3$
	$\mathbf{b}_3 = \frac{9}{20}\mathbf{p}_2 + \frac{11}{2}\mathbf{p}_3$		$\mathbf{b}_3 = \frac{9}{20}\mathbf{p}_2 + \frac{11}{20}\mathbf{p}_3$
	$\mathbf{b}_4 = \frac{3}{10}\mathbf{p}_2 + \frac{1}{2}\mathbf{p}_3 + \frac{1}{5}\mathbf{p}_4$		$\mathbf{b}_4 = \frac{3}{10}\mathbf{p}_2 + \frac{1}{2}\mathbf{p}_3 + \frac{1}{5}\mathbf{p}_4$
	$\mathbf{b}_5 = \mathbf{p}_5$		$\mathbf{b}_5 = \mathbf{p}_5$
$n = 6$	$\mathbf{b}_0 = \mathbf{p}_0$		$\mathbf{b}_0 = \mathbf{p}_0$
	$\mathbf{b}_1 = \frac{1}{6}\mathbf{p}_1 + \frac{1}{6}\mathbf{p}_2 + \frac{2}{6}\mathbf{p}_3$	$\times$	$\mathbf{b}_1 = \frac{1}{6}\mathbf{p}_1 + \frac{1}{6}\mathbf{p}_2 + \frac{2}{3}\mathbf{p}_3$
	$\mathbf{b}_2 = \frac{1}{15}\mathbf{p}_2 + \frac{6}{15}\mathbf{p}_3$	$\times$	$\mathbf{b}_2 = \frac{1}{15}\mathbf{p}_2 + \frac{14}{15}\mathbf{p}_3$
	$\mathbf{b}_3 = \frac{17}{10}\mathbf{p}_3$	$\times$	$\mathbf{b}_3 = \mathbf{p}_3$
	$\mathbf{b}_4 = 2\mathbf{p}_3 + \frac{1}{15}\mathbf{p}_4$	$\times$	$\mathbf{b}_4 = \frac{14}{15}\mathbf{p}_3 + \frac{1}{15}\mathbf{p}_4$
	$\mathbf{b}_5 = 2\mathbf{p}_3 + \frac{1}{6}\mathbf{p}_4 + \frac{1}{6}\mathbf{p}_5$	$\times$	$\mathbf{b}_5 = \frac{2}{3}\mathbf{p}_3 + \frac{1}{6}\mathbf{p}_4 + \frac{1}{6}\mathbf{p}_5$
	$\mathbf{b}_6 = \mathbf{p}_6$		$\mathbf{b}_6 = \mathbf{p}_6$
$n = 7$	$\mathbf{b}_0 = \mathbf{p}_0$		$\mathbf{b}_0 = \mathbf{p}_0$
	$\mathbf{b}_1 = \frac{1}{7}\mathbf{p}_1 + \frac{1}{7}\mathbf{p}_2 + \frac{3}{7}\mathbf{p}_3 + \frac{2}{7}\mathbf{p}_4$		$\mathbf{b}_1 = \frac{1}{7}\mathbf{p}_1+\frac{1}{7}\mathbf{p}_2+\frac{3}{7}\mathbf{p}_3+\frac{2}{7}\mathbf{p}_4$
	$\mathbf{b}_2 = \frac{1}{21}\mathbf{p}_2 + \frac{11}{21}\mathbf{p}_3 + \frac{3}{7}\mathbf{p}_4$		$\mathbf{b}_2 = \frac{1}{21}\mathbf{p}_2 + \frac{11}{21}\mathbf{p}_3 + \frac{3}{7}\mathbf{p}_4$
	$\mathbf{b}_3 = \frac{18}{35}\mathbf{p}_3 + \frac{17}{35}\mathbf{p}_4$		$\mathbf{b}_3 = \frac{18}{35}\mathbf{p}_3 + \frac{17}{35}\mathbf{p}_4$
	$\mathbf{p}_4 = \frac{17}{35}\mathbf{p}_3 + \frac{18}{35}\mathbf{p}_4$		$\mathbf{b}_4 = \frac{17}{35}\mathbf{p}_3 + \frac{18}{35}\mathbf{p}_4$
	$\mathbf{b}_5 = \frac{3}{7}\mathbf{p}_3 + \frac{11}{21}\mathbf{p}_4 + \frac{1}{21}\mathbf{p}_5$		$\mathbf{b}_5 = \frac{3}{7}\mathbf{p}_3 + \frac{11}{21}\mathbf{p}_4 + \frac{1}{21}\mathbf{p}_5$
	$\mathbf{b}_6 = \frac{2}{7}\mathbf{p}_3 + \frac{3}{7}\mathbf{p}_4 + \frac{1}{7}\mathbf{p}_5 + \frac{1}{7}\mathbf{p}_6$		$\mathbf{b}_6 = \frac{2}{7}\mathbf{p}_3+\frac{3}{7}\mathbf{p}_4+\frac{1}{7}\mathbf{p}_5+\frac{1}{7}\mathbf{p}_6$
	$\mathbf{b}_7 = \mathbf{p}_7$		$\mathbf{b}_7 = \mathbf{p}_7$

Theorem 1. *The Bézier control points, denoted by $\{\mathbf{b}_k\}_{k=0}^{n}$, of a DP curve of degree n can be given in terms of DP control points, denoted by $\{\mathbf{p}_i\}_{i=0}^{n}$, as follows:*

$$\mathbf{b}_k = \sum_{i=1}^{\lfloor \frac{n}{2} \rfloor - 1} \frac{k\binom{n-k}{n-i}}{n\binom{n-1}{n-i}} \cdot \mathbf{p}_i + \sum_{i=\lceil \frac{n}{2} \rceil + 1}^{n-1} \frac{n-k\binom{k}{i}}{n-i\binom{n}{i}} \cdot \mathbf{p}_i + \binom{n-k}{n} \cdot \mathbf{p}_k, \quad \text{if } k = 0, \tag{1}$$