Available online at www.sciencedirect.com
ScienceDirect
Tectonophysics 423 (2006) 115 – 123
TECTONOPHYSICS
www.elsevier.com/locate/tecto

Measuring multifractality in seismic sequences

Luciano Telesca *, Vincenzo Lapenna

Istituto di Metodologie per l'Analisi Ambientale, Consiglio Nazionale delle Ricerche, C.da S.Loja, 85050 Tito (PZ), Italy

Received 18 January 2005; accepted 25 March 2006
Available online 11 May 2006

Abstract

We investigated the multifractal structure of the interevent times between successive earthquakes that occurred in Umbria-Marche, which is one of the most seismically active areas of central Italy. We used the Multifractal Detrended Fluctuation Analysis (MF-DFA), which permits detection of multifractality in nonstationary series. Analyzing the time evolution of the multifractal behaviour of the seismic sequence, a loss of multifractality during the aftershocks is revealed.

Keywords: Multifractal Detrended Fluctuation Analysis; Earthquakes

1. Introduction

Marked point processes describe events that occur at random locations in space and time and are characterized by an intensity value. Earthquakes can be regarded as spatio-temporal point processes, marked by the magnitude. Their spatial (latitude, longitude, depth), temporal (occurrence instant) and energy (magnitude) parameters are featured by power-law behaviour, which implies self-similarity and absence of characteristic length-scale. The Gutenberg-Richter law states that the probability distribution of the released energy is a power-law (Gutenberg and Richter, 1944). The epicentres occur on a fractal-like distribution of faults (Kagan, 1992). The Omori's law states that the number of aftershocks, which follow a main event, decays as a power-law with exponent close to minus one (Utsu et al., 1995). The fractal behaviour revealed in these statistics could be considered as the end-product of a self-organized critical state of the Earth's crust, analogous to the state of a sandpile, which evolves naturally to a critical repose angle in response to the steady supply of new grains at the summit (Bak et al., 1988).

Recently, much work has been focused on the characterizing the temporal distribution of a seismic sequence. For completely random seismic sequences, the probability density function (pdf) of the interevent times follows an exponential decreasing form; this is typical of Poissonian processes, which are memoryless processes with all the events independent of each other. On the contrary, the interevent times are generally power-law distributed for time-clusterized sequences (Telesca et al., 1999, 2000 a,b), which are featured by time-correlation properties among the events. But the pdf of the interevent intervals is only one window into a point process, because it yields only first-order information and it reveals none about the correlation properties. Therefore, time-fractal second-order methods are necessary to investigate the temporal fluctuations of seismic sequences more deeply. The use of statistics like the Allan Factor (Allan, 1966), the Fano Factor (Lowen and Teich, 1995), the Detrended Fluctuation Analysis (DFA)

* Corresponding author. Tel.: +39 0971 427201; fax: +39 0971 427271.
E-mail address: ltelesca@imaa.cnr.it (L. Telesca).

0040-1951/$ - see front matter © 2006 Elsevier B.V. All rights reserved.
doi:10.1016/j.tecto.2006.03.023

(Peng et al., 1995), has allowed gaining more insight into the time dynamics of seismicity in terms of (i) discrimination between Poissonian and clusterized sequences (Telesca et al., 2001a,b), (ii) spatial variability of time-clustering behaviour (Telesca et al., 2001c, 2003a, 2004a,b; Telesca and Lapenna, 2004), and (iii) magnitude-variability of the property of time-clusterization (Telesca et al., 2002a; Telesca and Macchiato, 2004). All these measures are consistent with each other, so that we can define one scaling exponent that is sufficient to capture the time dynamics of a seismic process.

But one scaling exponent is sufficient to completely describe a seismic process under the hypothesis that this is monofractal. Monofractals are homogeneous objects, in the sense that they have the same scaling properties, characterized by a single singularity exponent (Stanley et al., 1996). The need for more than one scaling exponent can derive from the existence of a crossover time scale, which separates regimes with different scaling behaviours, suggesting e.g. different types of correlations at small and large time scales, thus leading to different types of time dynamics intrinsic in the same seismic sequence (Telesca et al., 2002a). Different values of the same scaling exponent could be required for different segments of the same seismic sequence, indicating a time variation of the scaling behaviour, relying to a time variation of the underlying dynamics (Telesca et al., 2001a). Typically an enhancement of the time-clustering is detected in correspondence to aftershocks (Telesca and Macchiato, 2004). Furthermore, different scaling exponents can be revealed for many interwoven fractal subsets of the sequence; in this case the process is not a monofractal but multifractal. A multifractal object requires many indices to characterize its scaling properties. Multifractals can be decomposed into many–possibly infinitely many–subsets characterized by different scaling exponents. Thus multifractals are intrinsically more complex and inhomogeneous than monofractals and characterize systems featured by a spiky dynamics, with sudden and intense bursts of high frequency fluctuations (Meneveau and Sreenivasan, 1991).

A seismic process can be considered as characterized by a fluctuating behaviour, with temporal phases of low activity interspersed between those where the density of the events is relatively large. This "sparseness" can be well described by means of the concept of multifractal.

Multifractality in earthquakes has been investigated in Enescu et al. (2005) concerning Vrancea seismicity. This study revealed two distinct scaling regimes: non homogeneous and multifractal at small scales, monofractal and close to Poissonian at large scales. Furthermore it was found that the multifractal behaviour at small scales (minutes–hours) is clearly an effect of the "short" aftershock sequences that occurred after some major Vrancea earthquakes. Scaling analysis of seismicity in the space–time–magnitude domain has been performed in Molchan and Kronrod (2005), pointing out to some evidence in favour of multifractality being present in seismicity.

The simplest type of multifractal analysis is given by the standard partition function multifractal formalism. By using this method aftershock-induced intermittent-type temporal fluctuations, interpreted in terms of multifractality, have been found in Irpinia (southern Italy) seismicity (Telesca et al., 2001d). But this method could suffer misleading results due to the presence of nonstationarity in the data. Another method based on the generalization of the Detrended Fluctuation Analysis (DFA) has been developed by Kantelhardt et al. (2002). This method, called Multifractal Detrended Fluctuation Analysis (MF-DFA) is able to reliably determine the multifractal scaling behaviour of nonstationary series.

2. Seismicity data

We study the sequence of 7521 earthquakes in a very seismically active area of central Italy, which was struck by a violent earthquake ($M_D = 5.8$) on September 26, 1997. The epicentre distribution of the events occurred from 1983 to 2003 is shown in Fig. 1 (data extracted from the instrumental catalogue of National Institute of Geophysics and Volcanology—INGV, http://www.ingv.it). The earthquakes are located in a circular area centered on the epicentre of the strongest M5.8 earthquake, with a radius of 100 km. The completeness magnitude, estimated after performing the Gutenberg-Richter analysis is 2.4.

3. Methods and data analysis

The main features of multifractals are to be characterized by high variability on a wide range of temporal or spatial scales, associated to intermittent fluctuations and long-range power-law correlations.

The interevent intervals examined in this paper present clear irregular dynamics (Fig. 2), characterized by sudden bursts of high frequency fluctuations, which suggest to perform a multifractal analysis, in order to evidence different scaling behaviours for different intensities of fluctuations. We applied the Multifractal Detrended Fluctuation Analysis (MF-DFA), which operates on the series $x(i)$, where $i = 1, 2, \ldots, N$ and N is the length of the series. With x_{ave} we indicate the mean value. We assume that $x(i)$ are increments of a random walk