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A heuristic iterated-subspace minimization method with pattern search for unconstrained optimization

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ABSTRACT

Recently, an increasing attention was paid on different procedures for an unconstrained optimization problem when the information of the first derivatives is unavailable or unreliable. In this paper, we consider a heuristic iterated-subspace minimization method with pattern search for solving such unconstrained optimization problems. The proposed method is designed to reduce the total number of function evaluations for the implementation of high-dimensional problems. Meanwhile, it keeps the advantages of general pattern search algorithm, i.e., the information of the derivatives is not needed. At each major iteration of such a method, a low-dimensional manifold, the iterated subspace, is constructed. And an approximate minimizer of the objective function in this manifold is then determined by a pattern search method. Numerical results on some classic test examples are given to show the efficiency of the proposed method in comparison with a conventional pattern search method and a derivative-free method.

1. Introduction

This paper is aimed to propose a method for solving the following unconstrained optimization problem:

$$\min_{x\in\mathbb{R}^n} f(x) \tag{1.1}$$

where f is assumed to be continuously differentiable and the information of the derivatives of $f(x)$ is unreliable or unavailable.

Pattern search methods are regarded as a kind of efficient algorithm for solving the derivative-free optimization problems due to their simplicity and usefulness. A lot of classical pattern search methods have been developed for unconstrained optimization such as multidirectional search of Dennis and Torczon [1], generalized pattern search method of Torczon [2], and grid-based framework of Coope and Price [3]. However, the implementation of these methods is relatively time consuming in comparison with the conventional derivative-based algorithms. This is because there are so many search directions in every pattern search process. To overcome this limitation, many researchers improved the pattern search methods by adopting some techniques in iterate acceptance criterion [4,5] or modifying the search direction set [6]. In the latter case, although positive bases [7] are utilized to replace bases in the pattern search methods, there are at least $n+1$ search directions in every round of pattern search. Hence, when a pattern search point (defined in Section 2) is found, we must implement at least $n+1$ times of function value estimations. In view of this, the implementation of the general pattern search methods may be inefficient particularly for high-dimensional problems. Consequently, to reduce the number

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of function evaluations, we consider implementing pattern search in a subspace with a lower dimension, which motivates us to integrate the iterated-subspace minimization methods into the pattern search framework.

A typical representation of previous work on the iterated-subspace methods is the method presented by Cragg and Levy [8]. They proposed a method to execute the minimization step in a k-dimensional subspace, which included the steepest descent direction. Shortly afterwards, Dennis and Turner [9] minimized a convex quadratic on a subspace by adding an extra vector which is independent on the existing subspace at each iteration. Then, Dixon et al. [10] constructed a 4-dimension subspace which was defined using the steepest descent direction, the Newton direction, and two other directions which were combinations of the two previous steps. Other related work about iterated-subspace minimization methods can also be found in Miele and Cantrell [11], Vinsome [12], Saad [13] and so on. Despite of the diversification for iterated-subspace method, it should be pointed out that the dimension of the "subspace" is lower than the original one. Taking advantage of this characteristic in the iterated-subspace method, the pattern search method can be improved to achieve a higher efficiency for implementation.

The combination of pattern search methods and iterated-subspace minimization methods can overcome the shortcoming of general pattern search methods. Instead of carrying out pattern search in original space, we implement pattern search in a lower-dimensional manifold, the iterated subspace. It is clearly that this technique can reduce the number of iterations and function evaluations.

The rest of this paper is organized as follows. In Sections 2 and 3, we provide a brief description of pattern search methods and iterated-subspace minimization methods. In the next section, we present framework of the new method together with some discussions on its new features. The corresponding convergence analysis is examined in Section 5. In Section 6, we give numerical results to demonstrate the feasibility and effectiveness of the proposed method. Finally, some concluding remarks are offered.

2. Generalized pattern search methods

Generalized pattern search methods for unconstrained optimization generate a sequence of iterates $\{x^{(k)}\}$ with non-increasing objective function values. At each iteration, the objective function is evaluated at a finite number of trial points on a grid [3] and the purpose is to look for one point which can yield a lower function value than the current iterate. If such a point is found, set it to be the new iterate, and the iteration is called successful; Otherwise, we declare it unsuccessful, record the current iterate (defined as a pattern search point [14]), refine the grid and update the trial points for the next iteration. Torczon [2] showed that generalized pattern search methods can produce a limit point for which the gradient of the objective function is zero. Coope and Price [3] proved the same conclusion from another point of view.

Now we can use a matrix $M^{(k)} \in \mathbb{R}^{n\times p^{(k)}}$ to indicate the set of trial directions on the kth iteration, where the columns of $M^{(k)}$ are the trial directions and $p^{(k)}$ is the number of trial directions. Δ_k is a rational scale factor. Then the pattern search process can be showed as follows:

Generalized Pattern Search:

Given initial iterate $x^{(0)} \in \mathbb{R}^n, f(x^{(0)}), M^{(0)} \in \mathbb{R}^{n\times p^{(0)}}$ and $\Delta^{(0)} > 0$,

While (Stopping conditions do not hold) do

Step 1. Find a step $s^{(k)} \in \Delta^{(k)}M^{(k)}$ by using Exploratory moves $(\Delta^{(k)}, M^{(k)})$.

Step 2. If $f(x^{(k)} + s^{(k)}) < f(x^{(k)})$, then $x^{(k+1)} = x^{(k)} + s^{(k)}$. Otherwise, $x^{(k+1)} = x^{(k)}$.

Step 3. Update $(\Delta^{(k)}, M^{(k)})$ to $(\Delta^{(k+1)}, M^{(k+1)})$, $k = k + 1$.

Remark 2.1. The trial search direction set $M^{(k)}$ usually contains a basis of $\mathbb{R}^n$ and its opposite direction. Recently, basis is replaced by positive basis [7] whose non-negative linear combinations span $\mathbb{R}^n$. Moreover, the number of positive basis' cardinality is between $n + 1$ and $2n$. For example, if V is a basis of $\mathbb{R}^n$, then

$$V_+ = [V, -Ve] \tag{2.1}$$

is a positive basis of $\mathbb{R}^n$, where $e = [1, 1, \ldots, 1]^{\mathrm{T}}$. Hence, the replacement can help to reduce the search directions.

3. Iterated-subspace minimization methods

Generally speaking, a prototype algorithm for iterated-subspace minimization methods can usually be expressed as follows:

Iterated-subspace minimization methods:

Given an initial iterate $x^{(0)}$:

Step 1. Stop with the current iterate $x^{(k)}$ if convergence tests are satisfied.

Step 2. Determine a full-rank subspace matrix $Z^{(k)} \in \mathbb{R}^{n\times z^{(k)}}$, where $z^{(k)} \ll n$.