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Attenuation, transport and diffusion of scalar waves in textured random media

L. Margerin *

Laboratoire de Géophysique Interne et Tectonophysique, Université Joseph Fourier/CNRS, BP 53, 38041, Grenoble, France

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Abstract

Most theoretical investigations of seismic wave scattering rely on the assumption that the underlying medium is statistically isotropic. However, deep seismic soundings of the crust as well as geological observations often reveal the existence of elongated or preferentially oriented scattering structures. In this paper, we develop mean field and radiative transfer theories to describe the attenuation and multiple scattering of a scalar wavefield in an anisotropic random medium. The scattering mean free path is found to depend strongly on the propagation direction. We derive a radiative transfer equation for statistically anisotropic random media from the Bethe–Salpeter formalism and propose a Monte Carlo method to solve this equation numerically. At longer times, the energy density is shown to obey a tensorial diffusion equation. The components of the diffusion tensor are obtained in closed form and excellent agreement is found between Monte Carlo simulations and analytical solutions of the diffusion equation. The theory has important potential implications for lithospheric models where scatterers are for example flat structures preferentially aligned along the surface. In this simple geometry, analytical expressions of the Coda Q parameter can be given explicitly in the diffusive regime. Our results suggest that pulse broadening and coda decay are controlled by different parameters, related to the eigenvalues of the diffusion tensor. These eigenvalues can differ by more than one order of magnitude. This theory could be applied to probe the anisotropy of length scales in the lithosphere.

Keywords: Multiple scattering; Radiative transfer; Random media; Anisotropy; Coda

1. Introduction

The propagation of seismic waves in heterogeneous media is a topic of continued interest for seismologists. Among the many approaches of the subject, stochastic theories have undergone a vigorous development in the last 20 years. The traditional range of application includes the modeling of the amplitude and phase of coherent arrivals, as well as the transport of the scattered energy. Important advances in the understanding of direct seismic wave attenuation due to scattering have been made by Sato (1982), Wu (1982), Shapiro and Kneib (1993), who developed new theories with specific application to seismic experiments. As an example, the travel-time corrected mean wave formalism of Sato (1982) has nicely reconciled known discrepancies between seismic measurements of attenuation and standard mean field theories. The study of seismic wave travel times in random media has also led to unexpected results. It has for example been shown that the first arrivals can propagate at velocities that are

* Tel.: +33 4 76 82 80 25; fax: +33 4 76 82 81 01.
E-mail address: Ludovic.Margerin@ujf-grenoble.fr.

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significantly higher than the volume-averaged velocity. This phenomenon has been termed 'velocity shift' and numerous extensive studies can be found in the literature (see, e.g. Roth et al., 1993; Shapiro et al., 1996).

Beyond a length scale known as the mean free path a large part of the energy of the coherent waves has been transferred to scattered waves which eventually form the coda of short period seismic events. The scattered waves transport the energy at distances larger than the mean free path and are responsible for the rapid field fluctuations, which are averaged out in the mean field approach. Thus, the information pertaining to the energy transport in a scattering medium is contained in the second statistical moment of the field. Field–field correlation theory is an extremely powerful tool that establishes the link between wave and radiative transfer equations and provides a theoretical background to explain many striking wave phenomena, such as electron's localization in condensed matter (Vollhardt and Wölfle, 1982), or coherent backscattering of optical waves (Akkermans et al., 1988). Correlation theory of random fields has also known some remarkable achievements in seismology with the development of coda wave interferometry (Snieder et al., 2002) or the Green function reconstruction from coda waves (Campillo and Paul, 2003). These methods make use of the phase information contained in multiply scattered waves to gain information about the medium. Such approaches go beyond the scope of this paper where attention will mostly be given to the energy density.

In seismology, the acoustic transport equation has been introduced by Wu (1985). Later, the theory has been extended to time-dependent cases by Zeng et al. (1991), Sato (1995), and to elastic waves including mode conversions by Zeng (1993) and Sato (1994). Data analysis tools such as the multiple-lapse-time-window analysis have been developed and applied in numerous regions of the world (e.g. Fehler et al., 1992; Hoshiba et al., 2001). Recent years have seen the emergence of even more realistic and challenging modelings including the possible depth dependence of background velocities and scattering properties (Margerin et al., 1998; Hoshiba et al., 2001; Lacombe et al., 2003), the interpretation of short period scattered waves at the global scale (Margerin and Nolet, 2003; Shearer and Earle, 2004) and the scattering of long period Rayleigh waves (Sato and Nishino, 2002). In most of these works, radiative transfer is introduced as a phenomenological theory, leaving a gap with the underlying wave equation. However, in condensed matter physics, radiative transfer emerges as a rigorous consequence of correlation theory, and this is the approach that will be adopted throughout the paper.

Although anisotropic media play a fundamental role in explaining many seismic observations such as anomalous splitting of core-sensitive normal modes or shear wave birefringence, most studies on multiple scattering of seismic waves assume implicitly that the underlying medium is statistically isotropic (see however the works of Iooss, 1998; Samuelides and Mukerji, 1998; Müller and Shapiro, 2003; Kravtsov et al., 2003). Yet, deep seismic soundings and geological maps often reveal elongated or laminated structures (see, e.g. Thybo, 2002). Thus, a scattering theory of anisotropic random media would be a useful tool to quantify the potential impact of textured geological structures on the wavefield. Important efforts have been invested in the study of wave propagation through anisotropic random media, mostly in optics and condensed matter physics. Furutsu (1980) proposed a phenomenological acoustic radiative transfer equation with broken rotational symmetry, and developed a diffusion approximation based on a diagonalization of the collision operator by perturbation theory. Anisotropic random media have also been studied by Wölfle and Bhatt (1984), in connection with the problem of electron localization. A stationary electromagnetic transport equation has been derived by Mischenko (2002) for arbitrarily shaped discrete scatterers using the Lax–Tversky's theory. A radiative transfer theory has also been proposed for electromagnetic waves propagating through nematic liquid crystals (van Tiggelen et al., 1996; Stark and Lubensky, 1997), and some experiments have been conducted (Wiersma et al., 2000; Johnson et al., 2002) that reveal an anisotropic transport of light.

The primary goal of the present work will be to investigate through a simple model, where mode conversions are neglected, how preferentially oriented and shaped inhomogeneities influence the transport of energy. We will employ the Dyson and Bethe–Salpeter formalism developed in condensed matter physics to quantify the attenuation of the mean field and derive from first principles a radiative transfer equation for scalar waves in anisotropic random media. We shall use the term 'anisomeric' random media since this refers to an isotropy of scale lengths. At longer times, the diffuse intensity will be shown to obey a tensorial diffusion equation that can be solved analytically. We will present numerical solutions of the anisotropic transfer equation and comparisons with diffusion theory. Finally, we will apply the diffusion theory to an anisomeric waveguide (for example the earth's crust) to show that broadening of seismogram envelopes and Coda Q may be used to