Computers and Mathematics with Applications 58 (2009) 436–448

Contents lists available at ScienceDirect

Computers and Mathematics with Applications

journal homepage: www.elsevier.com/locate/camwa

Permanence and stability in non-autonomous predator–prey Lotka–Volterra systems with feedback controls ☆

Linfei Nie [a,b,*], Zhidong Teng [a], Lin Hu [a], Jigen Peng [c]

[a] College of Mathematics and Systems Science, Xinjiang University, Urumqi 830046, China

[b] Department of Applied Mathematics, Xi'an Jiaotong University, Xi'an 710049, China

[c] Institute for Information and System Sciences, Research Center for Applied Mathematics, Xi'an Jiaotong University, Xi'an 710049, China

ARTICLE INFO

Article history:
Received 13 July 2007
Received in revised form 26 June 2008
Accepted 14 April 2009

Keywords:
Lotka–Volterra system
Predator-prey
Feedback control
Permanence
Lyapunov functional
Global stability

ABSTRACT

The main purpose of this article is considering whether or not the feedback controls have an influence on a non-autonomous predator–prey Lotka–Volterra type system. General criteria on permanence are established, which is described by an integral form and independent of some feedback controls. By constructing suitable Lyapunov functionals, a set of easily verifiable sufficient conditions are derived for the global stability of any positive solution to the model.

1. Introduction

Traditional two species autonomous or non-autonomous predator–prey Lotka–Volterra systems take the form

$$\begin{cases} \dfrac{\mathrm{d}x_1(t)}{\mathrm{d}t} = x_1(t)\big[b_1(t) - a_{11}(t)x_1(t) - a_{12}(t)x_2(t)\big], \\ \dfrac{\mathrm{d}x_2(t)}{\mathrm{d}t} = x_2(t)\big[-b_2(t) + a_{21}(t)x_1(t) - a_{22}(t)x_2(t)\big], \end{cases} \tag{1.1}$$

where $x_1(t)$ is the prey population density and $x_2(t)$ is the predator population density, $b_1(t)$, $a_{11}(t)$, the intrinsic growth rate and density-dependent coefficient of the prey, respectively; $b_2(t)$, $a_{22}(t)$, the intrinsic growth rate and density-dependent coefficient of the predator, respectively; $a_{12}(t)$ the capturing rate of the predator and $a_{21}(t)$ the rate of conversion of nutrients into the reproduction of the predator.

In the last decades, system (1.1) has been studied extensively, for example [1–9] and the references therein. Some sufficient conditions are obtained for the permanence, existence and uniqueness, and asymptotic stability of periodic solution for system (1.1).

However, we note that ecosystems in the real world are continuously distributed by unpredictable forces which can result in changes in the biological parameters such as survival rates. In ecology, we know that the practical question of interest is just whether or not an ecosystem can withstand those unpredictable disturbances which persist for a finite period of

☆ This work was supported by the National Natural Science Foundation of P.R. China (60764003), the Major Project of The Ministry of Education of P.R. China (207130), the Scientific Research Programmes of Colleges in Xinjiang (XJEDU2007G01, XJEDU2006I05) and the Natural Science Foundation of Xinjiang University(XY080103,BS080105).

* Corresponding author at: College of Mathematics and Systems Science, Xinjiang University, Urumqi 830046, China.
E-mail address: nielinfei@xju.edu.cn (L. Nie).

0898-1221/$ – see front matter © 2009 Elsevier Ltd. All rights reserved.
doi:10.1016/j.camwa.2009.04.014

time. In the language of control variables, we call the disturbance functions control variables. Whereas, the control variables discussed in much of the literature are constants or time dependent [10–12].

Recently, many scholars have done works on the ecosystem with feedback controls (see [13–20] and the references cited therein). In particular, Gopalsamy and Weng [21] discussed the asymptotic behavior of solutions in Logistic systems with feedback controls, Weng [22] considered a class of periodic integro-differential systems with feedback controls, Xiao [23] considered a two species competitive system with feedback controls, Chen [24] considered a non-autonomous Lotka–Volterra competitive system with feedback controls. These motivate us to consider the following non-autonomous predator–prey Lotka–Volterra system with feedback controls

$$\begin{cases} \dfrac{dx_1(t)}{dt} = x_1(t)\big[b_1(t) - a_{11}(t)x_1(t) - a_{12}(t)x_2(t) + c_1(t)u_1(t)\big] \\ \dfrac{dx_2(t)}{dt} = x_2(t)\big[-b_2(t) + a_{21}(t)x_1(t) - a_{22}(t)x_2(t) - c_2(t)u_2(t)\big] \\ \dfrac{du_1(t)}{dt} = f_1(t) - e_1(t)u_1(t) - d_1(t)x_1(t) \\ \dfrac{du_2(t)}{dt} = -e_2(t)u_2(t) + d_2(t)x_2(t). \end{cases} \tag{1.2}$$

In this paper, we study whether or not the feedback controls have an influence on the permanence of a positive solution of the general non-autonomous predator–prey Lotka–Volterra type systems, and establish the general criteria on the permanence of system (1.2), which is independent of some feedback controls. In additional, by constructing a suitable Lyapunov function, some sufficient conditions are obtained for the global stability of any positive solution to system (1.2).

This paper is organized as follows. In the next section, two useful lemmas, several basic assumptions for system (1.2) and the definitions of permanence are presented. We state and prove the sufficient conditions on the ultimately bounded and permanence of positive solutions for system (1.2), which is described by integrable form and independent of some feedback controls in Section 3. In the last section, a set of easily verifiable sufficient conditions are derived for the global stability of any positive solution of system (1.2).

2. Preliminaries

Let $R_+ = (0, \infty)$ and $R_{+0} = [0, \infty)$. In this section, we consider the following first order linear differential equations with a parameter

$$\frac{dv(t)}{dt} = g(t, \beta) - d(t)v(t), \tag{2.1}$$

where $g(t, \beta)$ is a continuous function defined on $(t, \beta) \in R_{+0} \times [0, \beta_0]$ and β_0 is a constant, $d(t)$ is a continuous function defined on R_{+0}. For system (2.1) we introduce the following assumptions.

(A_1) Function $g(t, \beta)$ is a non-negative bounded on $R_{+0} \times [0, \beta_0]$ and satisfies the Lipschitz condition with $\beta \in [0, \beta_0]$, i.e., there is a constant $L = L(\beta_0) > 0$ such that $|g(t, \beta_1) - g(t, \beta_2)| \le L|\beta_1 - \beta_2|$ for all $t \in R$, $\beta_1, \beta_2 \in [0, \beta_0]$.
(A_2) Function $d(t)$ is non-negative bounded on R_{+0} and there is a constant $\omega_1 > 0$ such that $\liminf_{t\to\infty} \int_t^{t+\omega_1} d(s)\,ds > 0$.

From assumptions (A_1) and (A_2), it is easy to proved that for any $(t_0, v_0) \in R_{+0} \times R_+$ and $\beta \in [0, \beta_0]$, system (2.1) has a unique solution $v_\beta(t)$ satisfying $v_\beta(t_0) = v_0$.

In system (2.1), when parameter $\beta = 0$ we obtain the following system

$$\frac{dv(t)}{dt} = g(t, 0) - d(t)v(t). \tag{2.2}$$

Let $v_\beta^*(t)$ be a fixed solution of system (2.1) defined on R_{+0}. We say that $v_\beta^*(t)$ is globally uniformly attractive on R_{+0}, if for any constants $\eta > 1$ and $\varepsilon > 0$ there is a constant $T = T(\eta, \varepsilon) > 0$ such that for $t_0 \in R_{+0}$ and any solution $v_\beta(t)$ of system (2.1) with $v_\beta(t_0) \in [\eta^{-1}, \eta]$, one has $|v_\beta(t) - v_\beta^*(t)| < \varepsilon$ for all $t \ge t_0 + T$. By Lemma 4 given in [1], we have

Lemma 2.1. *Suppose that assumptions* (A_1) *and* (A_2) *hold. Then,*

(a) *there is a constant $M > 0$ such that $\limsup_{t\to\infty} v_\beta(t) \le M$ for any positive solution $v_\beta(t)$ of system (2.1).*
(b) *each fixed solution $u_\beta^*(t)$ of system (2.1) is globally uniformly attractive on R_{+0}.*
(c) *if there is a constant $\omega_2 > 0$ such that $\liminf_{t\to\infty} \int_t^{t+\omega_2} g(s, \beta)\,ds > 0$ for all $\beta \in [0, \beta_0]$, then there is a constant $\eta > 1$ such that $\eta^{-1} \le \liminf_{t\to\infty} v_\beta(t) \le \limsup_{t\to\infty} v_\beta(t) \le \eta$ for any solution $v_\beta(t)$ of system (2.1).*

Let $v_0 \in R_+$, $t_0 \in R_{+0}$ and $\beta \in [0, \beta_0]$, and $v_\beta(t)$, $v_0(t)$ be the solutions of systems (2.1) and (2.2) with initial values $v_\beta(t_0) = v_0$ and $v_0(t_0) = u_0$, respectively. We can get the following result.

Lemma 2.2. *Suppose that assumptions* (A_1) *and* (A_2) *hold, then $v_\beta(t)$ converges to $v_0(t)$ uniformly for $t \in [t_0, \infty)$ as $\beta \to 0$.*