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Convergence performance comparison of quantum-inspired multi-objective evolutionary algorithms[☆]

Zhiyong Li [a,b,*], Günter Rudolph [b], Kenli Li [a]

[a] *School of Computer and Communication, Hunan University, Changsha, 410082, China*
[b] *Lehrstuhl für Algorithm Engineering, Universität Dortmund, 44221 Dortmund, Germany*

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ABSTRACT

In recent research, we proposed a general framework of quantum-inspired multi-objective evolutionary algorithms (QMOEA) and gave one of its sufficient convergence conditions to the Pareto optimal set. In this paper, two Q-gate operators, H_ϵ gate and $R\&N_\epsilon$ gate, are experimentally validated as two Q-gate paradigms meeting the convergence condition. The former is a modified rotation gate, and the latter is a combination of rotation gate and NOT gate with the specified probability. To investigate their effectiveness and applicability, several experiments on the multi-objective 0/1 knapsack problems are carried out. Compared to two typical evolutionary algorithms and the QMOEA only with rotation gate, the QMOEA with H_ϵ gate and $R\&N_\epsilon$ gate have more powerful convergence ability in high complex instances. Moreover, the QMOEA with $R\&N_\epsilon$ gate has the best convergence in almost all of the experimental problems. Furthermore, the appropriate ε value regions for two Q-gates are verified.

1. Introduction

For the capability of searching simultaneously the whole of solution spaces of the multi-objective optimization problem (MOP) using a population of feasible solutions based on stochastic mechanisms, evolutionary algorithms have more advantage in dealing with discontinuous and concave Pareto fronts than traditional mathematical programming techniques. A large number of multi-objective evolutionary algorithms (MOEA) that employ some innovative mechanisms have been proposed during the last two decades, such as MOGA [1], NPGA [2], NSGA2 [3] and SPEA2 [4] etc. The last two have become very popular evolutionary algorithms for multi-objective optimization problems (MOP) in the last few years. Some important theoretical work related to MOEA has been done. Rudolph has investigated convergence properties of some MOEAs under partially ordered finite set theory [5,6].

During the last decade, the quantum computational theory had attracted serious attention due to their remarkable superiority in computational and mechanical aspects, which was demonstrated in Shor's quantum factoring algorithm [7] and Grover's database search algorithm [8]. Integrating the quantum computing mechanisms and classical evolutionary algorithms, some quantum-inspired evolutionary algorithms (QEA) were proposed in [9–13], which are characterized by some quantum mechanics such as uncertainty, superposition, interference etc. Recently some quantum-inspired multi-objective evolutionary algorithms (QMOEA) combining MOEA with QEA were proposed to solve the multi-objective optimization problem (MOP) [14,15]. In [16] we discussed a general framework of QMOEA and presented one of its convergence conditions.

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* Corresponding author at: School of Computer and Communication, Hunan University, Changsha, 410082, China.
E-mail addresses: jt_lizhiyong@hnu.cn (Z. Li), guenter.rudolph@uni-dortmund.de (G. Rudolph).

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In this paper, we test the convergence performance of QMOEAs with several different Q-gate strategies on the multi-objective 0/1 knapsack benchmark problems. Two Q-gate strategies, H_ϵ gate and $R\&N_\epsilon$ gate, which satisfy the convergence condition are experimented and discussed. As the referenced algorithms, the popular MOEAs, NSGA2 and SPEA2, and the QMOEA with classic rotation gate are tested and evaluated simultaneously.

2. The general framework of QMOEA and its convergence condition

2.1. The general framework of quantum-inspired multi-objective evolutionary algorithms

Integrating the basic principle of quantum-inspired computing (QC) and general schemes of MOEA, we proposed a general framework of quantum-inspired multi-objective evolutionary algorithms in [16] as follows:

Procedure of the QMOEAs' Basic Framework

begin
  $t \leftarrow 0$
i) initialize the Q-population $Q(t)$
ii) the archives set $A(t) = \phi$
iii) while (not termination condition) do
  begin
    $t \leftarrow t + 1$
iv) make $P(t)$ by observing $Q(t-1)$
v) evaluate the solutions set $P(t)$
vi) rebuild $A(t)$ by selecting all of nondominated solutions from $A(t-1) \cup P(t)$
vii) extract objectives solutions set $O(t)$ from $A(t) \cup P(t)$
viii) make $Q(t)$ by updating $Q(t-1)$ according to $O(t)$ on one Q-gate strategy
  end
end

In above procedure, those basic principle of quantum-inspired evolutionary computation are similar to those QEAs [12,13] etc. Here we do not give the details. In MOEA some techniques are so successful that they have become general schemes, such as nondominated ranking, solutions diversity maintaining and elitism solutions archiving [17], and they are used in this framework. Especially, the elitists archives set $A(t)$ is crucial to its convergence property to the Pareto front [5].

2.2. One of the sufficient convergence conditions to QMOEAs

Now let us see one of convergence conditions for this framework. Firstly, let S be the feasible solution set for a MOP. f is the function mapping the variable set S to its image set and the comparing relation "$\preceq$" denotes "weak Pareto dominance" relation among those elements in the image set $f(S)$. According to those concepts and definitions about partially ordered set in [5,6], we can look upon the image space, $(f(S), \preceq)$, as a partially ordered set. Here the set $M_f(S, \preceq)$ denotes the set of minimal elements of the image space $f(S)$, which equals the Pareto optimal set of the MOP. By the construction of the basic framework, $A(t)$ is the archives solutions set. We define the concept *convergence to the Pareto optimal set with probability 1* as follows [6]:

Definition 2.1. Let $F^* = M_f(S, \preceq)$ and $A(t)$ be the archives solutions set of QMOEA. The QMOEA is said to converge with probability 1 to the Pareto optimal set if

$$\delta_{F^*}(f(A(t))) \to 0 \text{ with probability 1 as } t \to \infty.$$

Here the measure function $\delta_A(B) = |A| - |A \cap B|$ means the number of elements that are in the set A but not in the set B. The archives set $A(t)$ is defined by $M_f(A(t-1) \cup P(t), \preceq)$. Thus we can get one of the sufficient convergence conditions as follows:

Theorem 2.2 (*Sufficient Convergence Condition*). *Let S be a feasible solution set of MOP. One of the sufficient conditions by whose this QMOEA converges with probability 1 to its Pareto optimal set is that there exists a real number ϵ_0, $0 < \epsilon_0 < 1$, which satisfies* $\mathrm{Pro}(s \in P(t)) \geq \epsilon_0$ *for all $s \in S$, $t > 0$ and* $\mathrm{Pro}(s \in P(t))$ *is independent from each other for different t.*

Here $\mathrm{Pro}(s \in P(t))$ denotes the probability that the population $P(t)$ contains the solution s. The proofs for this theorem can be found in [18].

3. Three Q-gate strategies and their convergence properties

Based on the QMOEA's framework, we can use variant Q-gate strategies in viii) to construct a variety of algorithms. Now we discuss three Q-gate strategies and their convergence properties according Theorem 2.2.