Computers and Mathematics with Applications 57 (2009) 587–595

Contents lists available at ScienceDirect

Computers and Mathematics with Applications

journal homepage: www.elsevier.com/locate/camwa

Global solvability for a second order nonlinear neutral delay difference equation

Zeqing Liu [a], Yuguang Xu [b], Shin Min Kang [c,*]

[a] Department of Mathematics, Liaoning Normal University, P.O. Box 200, Dalian, Liaoning, 116029, People's Republic of China

[b] Department of Mathematics, Kunming Teacher's College, Kunming, Yunnan, 650031, People's Republic of China

[c] Department of Mathematics and the Research Institute of Natural Science, Gyeongsang National University, Jinju 660-701, Republic of Korea

ARTICLE INFO

Article history:
Received 6 January 2008
Received in revised form 16 July 2008
Accepted 10 September 2008

Keywords:
Second order nonlinear neutral delay difference equation
Uncountably many bounded nonoscillatory solutions
Contraction mapping

ABSTRACT

This paper studies the global existence of solutions of the second order nonlinear neutral delay difference equation

$$\Delta(a_n\Delta(x_n+bx_{n-\tau}))+f(n,x_{n-d_{1n}},x_{n-d_{2n}},\ldots,x_{n-d_{kn}})=c_n,\quad n\geq n_0$$

with respect to all $b\in\mathbb{R}$. A few results on global existence of uncountably many bounded nonoscillatory solutions are established for the above difference equation. Several nontrivial examples which dwell upon the importance of the results obtained in this paper are also included.

© 2008 Elsevier Ltd. All rights reserved.

1. Introduction and preliminaries

Recently, there has been increasing interest in the study of qualitative analysis of various second order difference equations, for example, see [1–12] and the references cited there.

Tang [9] discussed the existence of a bounded nonoscillatory solution for the second order linear delay difference equations

$$\Delta^2x_n=p_nx_{n-k},\quad n\geq 0,\tag{1.1}$$

$$\Delta^2x_n=\sum_{i=1}^{\infty}p_i(n)x_{n-k_i},\quad n\geq 0.\tag{1.2}$$

Zhang and Li [12] obtained some oscillation criteria for the second order advanced functional difference equation

$$\Delta(a_n\Delta x_n)+p_nx_{g(n)}=0.\tag{1.3}$$

Thandapani et al. [10] considered necessary and sufficient conditions for the asymptotic behavior of nonoscillatory solutions of the difference equation

$$\Delta(a_n\Delta x_n)=q_nx_{n+1},\quad n\geq 0,\tag{1.4}$$

and discussed a few sufficient conditions for the asymptotic behavior of certain types of nonoscillatory solutions of the second order difference equation

$$\Delta(a_n\Delta x_n)=q_nf(x_{n+1}),\quad n\geq 0.\tag{1.5}$$

* Corresponding author.
E-mail addresses: zeqingliu@sina.com.cn (Z. Liu), mathxu5329@163.com (Y. Xu), smkang@nongae.gsnu.ac.kr (S.M. Kang).

0898-1221/$ – see front matter © 2008 Elsevier Ltd. All rights reserved.
doi:10.1016/j.camwa.2008.09.050

Li and Zhu [6] established the asymptotic behavior of the second order nonlinear difference equation

$$\Delta(r_{n-1}\Delta x_{n-1}) + q_n(\Delta x_n)^\beta - p_n x_n^\alpha = e_n, \quad n \geq 0. \tag{1.6}$$

Cheng et al. [2] and Zhang [11] discussed the asymptotic behaviors of solutions and nonoscillatory solutions for some special cases of Eq. (1.6), respectively. Recently, Jinfa [3] utilized the contraction principle to study the existence of a nonoscillatory solution for the second order neutral delay difference equation with positive and negative coefficients

$$\Delta^2 (x_n + p x_{n-m}) + p_n x_{n-k} - q_n x_{n-l} = 0, \quad n \geq n_0 \tag{1.7}$$

under the condition $p \neq -1$. Migda and Migda [8] gave the asymptotic behavior of the second order neutral difference equation

$$\Delta^2 (x_n + p x_{n-k}) + f(n, x_n) = 0, \quad n \geq 1. \tag{1.8}$$

Very recently, Meng and Yan [7] investigated the sufficient and necessary conditions of the existence of the bounded nonoscillatory solutions for the second order nonlinear neutral delay difference equation

$$\Delta^2 (x_n - p x_{n-\tau}) = \sum_{i=1}^{m} q_i f_i(x_{n-\sigma_i}), \quad n \geq n_0. \tag{1.9}$$

However, to the best of our knowledge, neither did anyone investigate the global existence of nonoscillatory solutions for Eqs. (1.7)–(1.9) with respect to all $p \in \mathbb{R}$, nor did they discuss the existence of uncountably many bounded nonoscillatory solutions for Eqs. (1.1)–(1.9) and any other second order difference equations.

Motivated by the papers mentioned above, in this paper we investigate the following more general second order nonlinear neutral delay difference equation

$$\Delta\left(a_n \Delta(x_n + b x_{n-\tau})\right) + f(n, x_{n-d_{1n}}, x_{n-d_{2n}}, \ldots, x_{n-d_{kn}}) = c_n, \quad n \geq n_0, \tag{1.10}$$

where $b \in \mathbb{R}$, $\tau, k \in \mathbb{N}$, $n_0 \in \mathbb{N}_0$, $\{a_n\}_{n\in\mathbb{N}_{n_0}}$ and $\{c_n\}_{n\in\mathbb{N}_{n_0}}$ are real sequences with $a_n > 0$ for $n \in \mathbb{N}_{n_0}$, $\bigcup_{l=1}^{k}\{d_{ln}\}_{n\in\mathbb{N}_{n_0}} \subseteq \mathbb{Z}$, and $f : \mathbb{N}_{n_0} \times \mathbb{R}^k \to \mathbb{R}$ is a mapping. Using the contraction principle, we establish some global existence results of uncountably many bounded nonoscillatory solutions for Eq. (1.10) relative to all $b \in \mathbb{R}$. Our results sharp and improve Theorem 1 in [3]. To illustrate our results, seven examples are also included.

On the other hand, using similar arguments and techniques, the results presented in this paper could be extended to second order nonlinear neutral delay differential equations. Of course, we shall continue to study these possible extensions in the future.

Throughout this paper, we assume that Δ is the forward difference operator defined by $\Delta x_n = x_{n+1} - x_n$, $\Delta^2 x_n = \Delta(\Delta x_n)$, $\mathbb{R} = (-\infty, +\infty)$, $\mathbb{R}^+ = [0, +\infty)$, $\mathbb{Z}$ and $\mathbb{N}$ stand for the sets of all integers and positive integers, respectively,

$$N_a = \{n : n \in \mathbb{N} \text{ with } n \geq a\}, \quad Z_a = \{n : n \in \mathbb{Z} \text{ with } n \geq a\}, \quad a \in \mathbb{Z},$$
$$\alpha = \inf\{n - d_{ln} : 1 \leq l \leq k, n \in \mathbb{N}_{n_0}\}, \quad \beta = \min\{n_0 - \tau, \alpha\},$$
$$\lim_{n\to\infty} (n - d_{ln}) = +\infty, \quad 1 \leq l \leq k,$$

l_β^∞ denotes the Banach space of all bounded sequences on $\mathbb{Z}_\beta$ with norm

$$\|x\| = \sup_{n\in\mathbb{Z}_\beta} |x_n| \quad \text{for } x = \{x_n\}_{n\in\mathbb{Z}_\beta} \in l_\beta^\infty$$

and

$$A(N, M) = \left\{x = \{x_n\}_{n\in\mathbb{Z}_\beta} \in l_\beta^\infty : N \leq x_n \leq M,\ n \in Z_\beta\right\} \quad \text{for } M > N > 0.$$

It is easy to see that $A(N, M)$ is a bounded closed and convex subset of l_β^∞.

By a solution of Eq. (1.10), we mean a sequence $\{x_n\}_{n\in Z_\beta}$ with a positive integer $T \geq n_0 + \tau + |\alpha|$ such that Eq. (1.10) is satisfied for all $n \geq T$. As is customary, a solution of Eq. (1.10) is said to be oscillatory if it is neither eventually positive nor eventually negative. Otherwise, it is said to be nonoscillatory.

2. Existence of uncountable bounded nonoscillatory solutions

Now we investigate the existence of uncountable bounded nonoscillatory solutions for Eq. (1.10).

Download English Version:

https://daneshyari.com/en/article/469678

Download Persian Version:

https://daneshyari.com/article/469678

Daneshyari.com