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The lognormal distribution of metal resources in mineral deposits

Donald A. Singer *

10191 N Blaney Ave., Cupertino, CA 95014, United States

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ABSTRACT

For national or global resource estimation of frequencies of metals a lognormal distribution has sometimes been assumed but never adequately tested. Tests of frequencies of Cu, Zn, Pb, Ag, Au, Mo, Re, Ni, Co, Nb_2O_3, REE_2O_3, Cr_2O_3, Pt, Pd, Ir, Rh, and Ru, contents in over 3000 well-explored mineral deposits display a poor fit to the lognormal distribution. Neither a lognormal distribution nor a power law is an adequate model of the metal contents across all deposits. When these metals are grouped into 28 geologically defined deposit types, only nine of the over 100 tests fail to be fit by the lognormal distribution, and most of those failures are in two deposit types suggesting problems with those types. Significant deviations from lognormal distributions of most metals when ignoring deposit types demonstrate that there is not a global lognormal or power law equation for these metals. Mean and standard deviation estimates of each metal within deposit types provide a basis for modeling undiscovered resources. When tracts of land permissive for specific deposit types are delineated, deposit density estimates and contained metal statistics can be used in Monte Carlo simulations to estimate total amounts of undiscovered metals with associated explicit uncertainties as demonstrated for undiscovered porphyry copper deposits in the Tibetan Plateau of China.

1. Introduction

Whether considering a country's possible future supply of minerals, considering the merits of exploring for certain kinds of mineral deposits, or examining possible global availability of some mineral materials, having a probability distribution of the amounts of minerals of interest would be invaluable (Singer and Menzie, 2010). One probability distribution that has been suggested as appropriate for mineral resources is the lognormal distribution. Recommendations of the lognormal distribution as an appropriate model of the frequency of ore deposits have waxed and waned over the years. Part of the change in views is due to variation in the apparent fit of the distribution to case studies and part of the change may be due to variation in popularity of different techniques over time.

Much of the early research focused on the distribution of grades of mineral deposits or geochemical abundances (Ahrens, 1954; Matheron, 1959; Rasumovsky, 1940). These studies found an empirical and a theoretical basis for believing that the lognormal distribution is an appropriate model for observed mineral deposit grades and values of trace elements in samples. Usefulness of the lognormal distribution was further documented by the development of its theoretical foundations and by the empirical evidence of its applicability in biology, sociology, astronomy and economics provided by Aitchison and Brown (1963).

Based on studies of mineral production, Allais (1957) selected a lognormal distribution to represent the values of mineral deposits thereby suggesting a lognormal distribution of metals. Slichter et al. (1962) used a graphical fit of the lognormal distribution to the gross values of copper, lead, zinc, gold and silver mines in part of the Southwest of the United States. Gross values of sandstone-hosted uranium deposits in the Ambrosia Lake region of the United States were tested and shown to be well represented by the lognormal distribution (Griffiths and Singer, 1973). Economic effects on the fit of the lognormal distribution to diamond production were demonstrated by Sharp (1976). Zhang et al. (2004) found that copper equivalent grades of deposits in China could be represented by a lognormal distribution but metal content could only be represented by lognormal after separating the deposits and districts into different groups. Singer (1993) tested the distribution of ore tonnages and average grades of sixty-seven types of mineral deposits and found that most were not significantly different than the lognormal. Only five of the sixty-seven tonnages of ore distributions were significantly different than lognormal at the 1% level. Although it is commonly assumed that the distribution of metal amounts can be represented by lognormal distributions, the idea has seen little actual testing and few modern estimates of the parameters of these lognormal distributions of metals have been published. The major exception is an earlier study by Singer (2011) in which nine metal contents in 19 geologically defined deposits types were examined for lognormality. This study significantly broadens that analysis by adding nine metals, 11 deposit types and over 1000 mineral deposits, and it shows how estimates of the parameters can be used to assess undiscovered metal resources.

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* Tel.: +1 408 255 2687.
E-mail address: singer.finder@comcast.net.

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Before testing the distributions, consideration of the nature of mineral deposit data suitable for testing and the sources of these data are presented. In this study the ability of the lognormal distribution to fit the observed distribution of many metals is tested. In addition, estimates of the parameters of the lognormal distributions are provided where appropriate. Finally, an example is presented of using the lognormal distribution of copper content and the density of porphyry copper deposits in an assessment of undiscovered deposits in the Tibetan Plateau of China.

2. Mineral deposit information

Typically, a lognormal distribution can be used to model the observed distributions of homogeneous populations of variables representing weights, lengths, volumes, and grades of trace quantities. It tends to not fit grade distributions of elements that have grades greater than about 10%, such as Fe, Mn, and Al. So what is the proper homogeneous population that should be sampled to represent metals in mineral deposits?

The geologic and mining literature contains many terms such as district, zone, ore body, lens, shaft, vein, bench, open-pit, underground, and mine that might be considered as possible sampling units. These terms are applied in different ways by different groups at different points in time, making them undesirable as our sampling unit. Grade-and-tonnage data are available to varying degrees for districts, deposits, mines, and shafts. In many cases, old production data are available for some deposits and recent resource estimates are available for other deposits. A common error is mixing deposits with only old production data with deposits with modern complete resource data. Ideally past production data would be combined with complete resource estimates for each deposit used in the analysis. It is extremely important that all data used in the model represent the same sampling unit because mixing data from deposits with only old production with deposits with recent resource estimates usually produces bimodal distributions representing non-homogeneous populations and it may introduce correlations among the variables that are artifacts of the mixed sampling units. Models constructed using data from mixed sampling units are of questionable value because the frequencies observed are directly related to the proportion of deposits from each sampling unit and are unlikely to be representative of the proportion in the undiscovered deposits being estimated.

For the analysis here of the frequency distributions of metal contents, data used in grade-and-tonnage models is used because they were specifically prepared for assessments to show the frequencies of different sizes and grades of each mineral deposit type based on data collected on thousands of well-explored deposits from around the world. For each deposit type, these models help define a deposit, as opposed to a mineral occurrence or a weak manifestation of an ore-forming process. Data utilized to construct these models include average grades of each metal or mineral commodity of possible economic interest and the associated tonnage based on the total production, reserves, and resources at the lowest possible cutoff grade. These data represent an estimate of the endowment of each of many known deposits. Well-explored in this report means completely drilled in three dimensions or completely mined out. Additionally these data were gathered using spatial rules in order to be consistent in what they represent.

3. Data sources and spatial rules

For sediment-hosted zinc-lead deposits (Singer et al., 2009), all mineralized rock or alteration within 2 km was combined into one deposit for these deposits. Thus, if the alteration zones of two deposits are within 2 km of each other, they were combined. The two-kilometer rule was developed to try to insure that deposits in grade and tonnage and spatial density models correspond to deposits as geologic entities. Rules such as the two-kilometer rule are essential in order to have an internally consistent assessment system where the estimate of number of undiscovered deposits is consistent with the grade and tonnage model. Sediment-hosted zinc-lead types include: skarns and polymetallic replacements and some sedimentary-exhalative Zn–Pb deposits are grouped here as carbonate-hosted igneous deposits (CAig), Mississippi Valley Zn–Pb and some sedimentary-exhalative Zn–Pb deposits are classed as carbonate-hosted amagmatic here (CAam), some sedimentary-exhalative Zn–Pb deposits classed as carbonate-hosted metamorphic (CAme), most sedimentary-exhalative Zn–Pb (SEDEX) deposits are classed as shale-hosted amagmatic (SHam), Kipushi deposits classed as Kipushi, some sedimentary-exhalative Zn–Pb deposits classed as shale-hosted igneous (SHig), some sedimentary-exhalative Zn–Pb deposits classed as mixed lithology-hosted igneous (MLig), some sedimentary-exhalative Zn–Pb deposits classed as mixed lithology-hosted metamorphic (MLme), and sandstone-hosted Pb deposits (SSPb).

For the porphyry copper deposits used in this analysis (Singer et al., 2008), all mineralized rock or alteration within 2 km was combined into one deposit. Thus if the alteration zones of two deposits are within 2 km of each other, they were combined. Grades of rhenium in 56 of the porphyry copper deposits were gathered from a variety of sources for this study (Berzina et al., 2004; Northern Dynasty Minerals, 2011; Northisle, 2013).

Data on quantities of contained molybdenum from Climax-type porphyry deposits and low-fluorine stockwork molybdenum deposits were gathered from Ludington and Plumlee (2009) and Ludington et al. (2009), respectively.

Sediment-hosted copper deposits (Cox et al., 2003) were combined into one deposit if they were within 2 km of each other. Iron oxide Cu–Au deposits (Cox and Singer, 2007) were combined if they occurred within 2 km of each other.

For the deposits in the carbonatite model (Berger et al., 2009), the following rule was used to determine which ore bodies were combined. All mineralized rock or altered rock within 2 km was combined into one deposit. Some examples of deposits that were combined illustrate the effects of the application of this rule to combined deposits: (1) Salitre I and II deposits in Brazil and (2) Upper Fir and Fir in Canada.

Ni laterite bodies were combined into single Ni–Co deposits for all mineralized rock within 2 km (Berger et al., 2011). All chromite pods within 100 m of each other, measured from their margins, were merged into single deposits (Mosier et al., 2012).

For volcanogenic massive sulfide deposits (Mosier et al., 2009), the following spatial rule was used to determine which ore deposits were combined. All mineralized rock within 500 m was combined into one deposit. A 500-meter rule was used for volcanogenic massive sulfide deposits because of their smaller size and the scarcity of mapped alteration zones around these deposits. For example, in this report, Horne and Quemont in Quebec, Canada, are combined into one deposit, and Jerome in Arizona, United States, has been treated as two separate deposits, United Verde and United Verde Extension, because of the 500-meter rule. The volcanogenic massive sulfide deposits were classed into three types. The felsic type (VMSFel) includes those volcanogenic massive deposits hosted in dominantly felsic or bimodal-felsic rocks. The bimodal-mafic type (VMS-Bi) includes those volcanogenic massive sulfide deposits dominantly hosted in mafic volcanic rocks with rhyolite to dacite constituting 10 to 40% of the host rocks. The mafic type (VMSMaf) of volcanogenic massive sulfide deposit is dominantly hosted in mafic volcanic rocks and associated pelitic rocks.

A total of 123 deposits with reported grades and tonnages of sediment-hosted gold deposits were divided into a Carlin subtype (SedAuCar) and a Chinese subtype (SedAuChi) by Berger et al. (2013). A two hundred meter rule was used to combine adjacent mineralized bodies into one deposit for these subtypes. Tonnages of