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An efficient regularized inversion approach for self-potential data interpretation of ore exploration using a mix of logarithmic and non-logarithmic model parameters

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ABSTRACT

A very fast and efficient approach to self-potential (SP) data inversion for ore exploration has been developed. This approach is based on Tikhonov regularization and the conjugate gradient method, and simultaneously inverts for the depth (z), electric dipole moment (k), and angle of polarization (θ) of a buried anomalous body from SP data measured along a profile. This inversion algorithm works iteratively, and solves for z and k in the logarithmic-space ($\log(z)$ and $\log(k)$), and solves for θ in the linear-space (non-logarithmic). It is found that the original inversion formulation that uses the model parameters themselves (z, k and θ) is unstable and divergent. It is also found that the inversion formulation that uses the logarithm of the model parameters ($\log(z)$, $\log(k)$ and $\log(\theta)$) is unstable and divergent. Rather, the new inversion scheme that is based on the aforementioned mixed log-linear combination of the model parameters ($\log(z)$, $\log(k)$, and θ) overcomes and eliminates the mentioned instability and divergence problems. The sensitivity analysis and numerical experiments investigated have indicated that the new approach has a far better and far more optimized minimization search direction. This proposed technique fits the observed data by some geometrically simple body in the restricted class of vertical cylinder, horizontal cylinder, and sphere models. The applicability of the algorithm has been demonstrated on various reliable synthetic data sets with and without noise. The algorithm has been carefully and successfully applied to six real data examples, with ore bodies buried in different complex geologic settings and at various depths in the subsurface. The method is shown to be highly applicable for mineral exploration, and is of particular value in cases where the SP observed data is due to ore body embedded in the subsurface. On average, it took about 40 s of computation (not CPU) time on a 1 GHz PC.

1. Introduction

Self-potential (SP) methods measure naturally occurring electrical potentials in the underground, and have significant applications in mineral prospecting (e.g., Essa et al., 2008; Fedi and Abbas, 2013; Goldie, 2002; Nishi and Ishido, 2012; Peksen et al., 2011; Yüngül, 1950). In addition, these techniques have various applications in groundwater monitoring (e.g., Linde et al., 2007), and geotechnical engineering and environmental problems, for example, monitoring the leaking areas in water dams, mapping the water flow in subglacial drainage conditions, and geothermal investigation (e.g., Ikard et al., 2012; Kulessa et al., 2003; Onizawa et al., 2009). SP anomalies that occur over massive and disseminated sulfides mineral deposits are those of principal interest in mineral exploration (Agarwal and Srivastava, 2009; Corry, 1985; Corry et al., 1982). Various mechanisms addressing the origins of SP anomalies have been discussed by many authors (e.g., Kilty, 1984; Ishido, 2004; Patella, 1997a,b; Revil et al., 2001; Sato and Mooney, 1960; Sill, 1983; Stoll et al., 1995).

Recently, there has been renewed interest in ways of interpreting SP data, and various methods for this purpose have already been developed. Two- (2D) and three-dimensional (3D) SP inversion schemes solve for the source current density of the subsurface (Mendonca, 2008; Minsley et al., 2007). Though these schemes are linear with respect to the source current density (model parameters we seek to invert for), they, however, demand accurate *a priori* information about the electrical conductivity distribution in the subsurface to produce meaningful inversion results (Colangelo et al., 2006; Jardani and Revil, 2008; Jardani et al., 2007; Mehanee et al., 2011). The conductivity distribution can be obtained by solving alternative full 2D and 3D inverse problems (e.g., electromagnetic and/or direct current (dc) resistivity) which are nonlinear with respect to the conductivity we seek (Mehanee and Zhdanov, 2002; Mehanee et al., 2011).

Heiland (1940), Banerjee (1971) and Fitterman (1979) introduced methods based on nomograms and standardized curves. Bhattacharya et al. (1981), and Babu and Rao (1988) developed procedures for SP inversion based on a few characteristic points and distances. The

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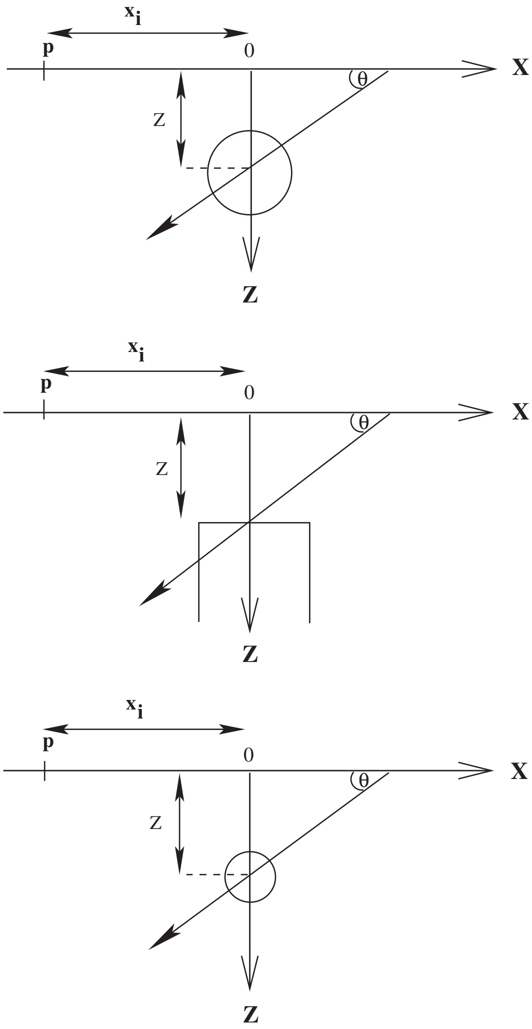

Fig. 1. A sketch showing cross-sectional views (after El-Araby et al., 2004), geometries and parameters of a sphere (top panel), a semi-infinite vertical cylinder (middle panel), and an infinitely long horizontal cylinder (bottom panel).

drawback with these approaches is that they are highly subjective, and hence can lead to major errors in the estimated SP parameters of the buried structure (Abdelrahman et al., 2003; El-Araby et al., 2004; Mehanee et al., 2011; Srivastava and Agarwal, 2009).

Abdelrahman et al. (2003, 2004, 2006) developed numerical techniques for successive inversions, which use the entire observed SP data only when inverting for the first parameter (e.g., depth), and then use a few characteristic data points out of the entire measured SP profile when inverting for the rest of the polarization parameters (e.g., the polarization angle and the electric dipole moment). It is noteworthy to mention that some methods (e.g., Abdelrahman et al., 1997) work under the assumption that the shape factor of the buried structure is known *a priori* or assumed. El-Araby et al. (2004) also developed a successive method to invert for the shape, depth, polarization angle and electric dipole moment of the anomalous body. However, his method uses the entire measured data when inverting only for the shape of the anomalous structure, and then uses only a few characteristic data points when inverting for the subsequent three model parameters. Srivastava and Agarwal (2009) developed an approach based on the concept of Enhanced Local Wave (ELW) number to interpret SP anomalies along a profile traversed over 2D and 3D sources, and insightfully applied and analyzed the approach to four field examples, with different levels of complexity, from the literature. Although their presented technique provides correct horizontal location of the source, they concluded that, for very shallow or outcropping anomalous sources, their technique neither identifies the nature of source geometry nor provides correct depth. They also reported in the case of noisy data that models of line of poles encountered some errors in the computed depth and structure index.

Patella (1997a,b) introduced a new imaging technique based on a cross correlation algorithm for the interpretation of SP data. Revil et al. (2001) developed a 2D imaging technique, based on a dipole source, that uses the cross correlation between two theoretical scanning functions. The possible occurrence of this dipole source within the subsurface is imaged by computing a probability function from the aforementioned cross correlation. However, this method demands *a priori* knowledge of the 2D electrical conductivity distribution in the subsurface. Peksen et al. (2011) used the particle swarm optimization (PSO) method for the interpretation of SP anomalies by some geometrically idealized bodies. They successfully applied this approach to four field examples from mining geophysics, and concluded that the method is robust, but can be affected by the noise level to some degree. Gokturkler and Balkaya (2012) described three global optimization approaches based on stochastic algorithms (genetic algorithm (GA), simulated annealing (SA), and particle swarm optimization (PSO)) to invert SP anomalies by some polarized bodies of simple geometries, and concluded that the inverse parameters recovered by the GA, SA and PSO are in good agreement. Fedi and Abbas (2013) developed a fast imaging technique (the so-called depth from extreme points (DEXP) method) based on the upward continuation to interpret SP anomalies.

The objective of this paper is to develop an efficient and rigorous deterministic inversion algorithm that is capable of both inverting the entire SP data set measured along a profile, rather than just a few characteristic data points out of this profile, produced by some ore body embedded in the subsurface, and simultaneously retrieving all the inverse parameters (the depth (z), electric dipole moment (k), and angle of polarization (θ)) of the causative body. The developed algorithm employs Tikhonov regularization, and utilizes the regularized conjugate gradient method to solve the minimization problem of the objective (parametric) functional. This inversion algorithm works iteratively and solves for z and k in the logarithmic-space (log-space, $log(z)$ and $log(k)$), and for θ in the linear-space (non-logarithmic-space). The aforementioned mixed log-linear combination of the model parameters ($log(z)$, $log(k)$, and θ) was essential and proved crucial in order to achieve the convergence and stability of the inverse solution throughout the iterative scheme of the minimization problem.

While there have been numerous successful applications of the Tikhonov regularization (Tikhonov and Arsenin, 1977), and the conjugate gradient method in various geophysical data inversions (e.g., Mehanee and Zhdanov, 2002; Tarantola, 1987; Wannamaker and Doerner, 2002; Zhdanov, 2002), I found for this particular SP inverse problem (as shall be seen and discussed in the numerical modeling, inversion, and discussion sections) that the inversion scheme that uses the model parameters themselves (z, k and θ) in its objective functional subject to minimization is unstable and non-convergent. It is also found that the inversion scheme that uses the logarithm of all the model parameters ($log(z)$, $log(k)$ and $log(\theta)$) is unstable and non-convergent. The heart of this paper, therefore, is the simultaneous and combined use of the log-space and linear-space of the model parameters ($log(z)$, $log(k)$ and θ) to overcome the instability and convergence problems, and hence solving this inverse problem simultaneously for the depth, dipole moment, and angle of polarization of a buried anomalous body from the SP data measured along a profile.

This approach fits the observed data by a class of some geometrically simple anomalous bodies in the restricted class of vertical cylinder, horizontal cylinder, and sphere models. The rationale behind the proposed scheme is that some geological settings can have an individual SP anomaly which can be inverted as a single polarized body. Consequently, fast and accurate quantitative inversion methods based on idealized bodies are useful as they can yield approximate interpretation by determining the depth and polarization parameters of the body (in the restricted