Computers and Mathematics with Applications 56 (2008) 2550–2555

Contents lists available at ScienceDirect

Computers and Mathematics with Applications

journal homepage: www.elsevier.com/locate/camwa

Genetic algorithm based on simplex method for solving linear-quadratic bilevel programming problem☆

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ARTICLE INFO

Article history:
Received 24 April 2008
Accepted 27 May 2008

Keywords:
The linear-quadratic bilevel programming problem
Genetic algorithm
Simplex method
Fitness value

ABSTRACT

The bilevel programming problems are useful tools for solving the hierarchy decision problems. In this paper, a genetic algorithm based on the simplex method is constructed to solve the linear-quadratic bilevel programming problem (LQBP). By use of Kuhn–Tucker conditions of the lower level programming, the LQBP is transformed into a single level programming which can be simplified to a linear programming by the chromosome according to the rule. Thus, in our proposed genetic algorithm, only the linear programming is solved by the simplex method to obtain the feasibility and fitness value of the chromosome. Finally, the feasibility of the proposed approach is demonstrated by the example.

1. Introduction

The bilevel programming problems are nested optimization problems with two levels in a hierarchy, the upper level and lower level decision-makers who have their own objective functions and constraints. The decision maker at the lower level (the follower) has to optimize its own objective function under the given parameters from the decision maker at the upper level (the leader), who, in return, selects the parameters so as to optimize its own objective function, with complete information on the possible reactions of the follower.

The bilevel programming problem(BLP) is defined as [1]:

$$\begin{aligned} &\min_x F(x,y) \\ &\text{subject to} \quad G(x,y) \le 0 \\ &\qquad\qquad\quad \min_y f(x,y) \\ &\qquad\qquad\quad \text{subject to } g(x,y) \le 0 \end{aligned} \tag{1}$$

where $F, f : R^{n_1} \times R^{n_2} \longrightarrow R$ are called the objective functions of the leader and the follower, respectively. $G : R^{n_1} \times R^{n_2} \longrightarrow R^p$ and $g : R^{n_1} \times R^{n_2} \longrightarrow R^q$ are the constraints of the leader and the follower, respectively. $x \in R^{n_1}$, $y \in R^{n_2}$ are the decision variables of the leader and the follower, respectively.

☆ Supported partly by the National Natural Science Foundation of China(Nos. 70771080, 60574071) and the Research Foundation for Outstanding Young Teachers, China University of Geosciences (Wuhan) (Nos. CUGQNW0801).

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doi:10.1016/j.camwa.2008.05.006

Although the designation bilevel and multilevel programming was firstly used by Candler and Norton [2], Bracken and McGill [3] gave the original formulation for bilevel programming in 1973. However, till 1980s, these problems started receiving the attention motivated by the game theory [4]. And many authors studied bilevel programming intensively and contributed themselves into those fields. Some surveyed the bilevel programming by presenting both theoretical results as well as solution approaches and a large number of applications [5–11], while some researched those problems in monographes [1,12–14]. Various approaches developed for the bilevel programming problems can be classified into the following categories [14]: extreme point algorithms mainly for the linear bilevel programming, branch-and-bound approach, complementary pivot approach, descent approach, penalty function approach and intelligent computation.

However, the bilevel programming is neither continuous anywhere nor convex even if the objective functions of the upper level and lower level and the constraints are all linear because the objective function of the upper level, generally speaking, is neither linear nor differentiable, because it is decided by the solution function of the lower level problem. Bard proved that the bilevel linear programming is a NP-Hard problem [15] and even it is a NP-Hard problem to search for the locally optimal solution of the bilevel linear programming [16]. So, it is greatly difficult to solve the bilevel programming for its non-convexity and non-continuity, especially the non-linear bilevel programming problem.

This paper considers the linear-quadratic bilevel programming problem(LQBP) where the lower level objective function is a convex quadratic and all remaining functions are linear. The remaining of the paper is organized as follows: The model and definitions of linear-quadratic bilevel programming problem is presenter in Section 2; Section 3 describes the genetic algorithm approach for solving LQBP; Some examples are illustrated to demonstrate the feasibility and efficiency in Section 4; Finally, the paper is concluded in Section 5.

2. The concepts and properties of the LQBP

When the objective function of the lower level is convex quadratic and all remaining functions are linear, the linear-quadratic bilevel programming problem can be formulated as follows:

$$\begin{aligned}(LQBP)\quad & \min_x F(x,y) = a^{\mathrm{T}}x + b^{\mathrm{T}}y \\ & \text{where } y \text{ solves} \\ & \min_y f(x,y) = c^{\mathrm{T}}x + d^{\mathrm{T}}y + (x^{\mathrm{T}}, y^{\mathrm{T}})Q(x^{\mathrm{T}}, y^{\mathrm{T}})^{\mathrm{T}} \\ & s.t.\ Ax + By \le r \\ & \qquad x, y \ge 0\end{aligned} \tag{2}$$

where $F(x,y)$, $f(x,y)$ are the objective functions of the leader and the follower, respectively. $a, c \in R^{n_1}$, $b, d \in R^{n_2}$, $A \in R^{m\times n_1}$, $B \in R^{m\times n_2}$, $r \in R^m$.$Q \in R^{(n_1+n_2)\times(n_1+n_2)}$ is a symmetric positive semi-definite matrix. $x \in R^{n_1}$, $y \in R^{n_2}$ are the decision variables under the control of the leader and the follower, respectively.

Definition 2.1. The constraint region of LQBP:

$$S = \{(x,y)|Ax + By \le r, x, y \ge 0\}.$$

Definition 2.2. The projection of S onto the leader's decision space:

$$S(X) = \{x \ge 0|\text{there exists a } y \text{ , such that } (x,y) \in S\}.$$

In order to ensure that the problem (2) is well posed we make assumption that S is non-empty and bounded.

In the problem (2), let

$$Q = \begin{bmatrix} Q_2 & Q_1^{\mathrm{T}} \\ Q_1 & Q_0 \end{bmatrix}$$

where $Q_0 \in R^{n_2\times n_2}$, $Q_1 \in R^{n_2\times n_1}$, $Q_2 \in R^{n_1\times n_1}$. Then $f(x,y)$ is transformed into

$$f(x,y) = c^{\mathrm{T}}x + x^{\mathrm{T}}Q_2x + (d + 2Q_1x)^{\mathrm{T}}y + y^{\mathrm{T}}Q_0y.$$

Note that $c^{\mathrm{T}}x + x^{\mathrm{T}}Q_2x$ is constant for each fixed $x \in S(X)$, we can assume $c = 0$, $Q_2 = 0$ to ignore those terms without loss of generality when solving the lower level programming. Thus the optimal solution to the follower problem can be obtained by solving the following problem:

$$\begin{aligned}& \min_y f(x,y) = (d + 2Q_1x)^{\mathrm{T}}y + y^{\mathrm{T}}Q_0y \\ & s.t.\ By \le r - Ax \\ & \qquad y \ge 0.\end{aligned} \tag{3}$$

Because Q_0 is a symmetric positive semi-definite matrix from the supposition that Q is a symmetric positive semi-definite matrix, there exists a unique and global solution, denoted by $y(x)$, to the problem (3) for each fixed $x \in S(X)$ [17].