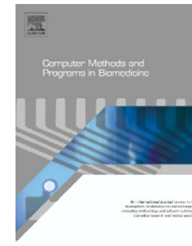
COMPUTER METHODS AND PROGRAMS IN BIOMEDICINE 84 (2006) 135–145



journal homepage: www.intl.elsevierhealth.com/journals/cmpb

Robust segmentation of cerebral arterial segments by a sequential Monte Carlo method: Particle filtering

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ARTICLE INFO

Article history:
Received 3 May 2006
Received in revised form
2 September 2006
Accepted 3 September 2006

Keywords:
Segmentation
Vessel tracking
CTA
Particle filter
Cerebral arteries

ABSTRACT

In this paper a method to extract cerebral arterial segments from CT angiography (CTA) is proposed. The segmentation of cerebral arteries in CTA is a challenging task mainly due to bone contact and vein contamination. The proposed method considers a vessel segment as an ellipse travelling in three-dimensional (3D) space and segments it out by tracking the ellipse in spatial sequence. A particle filter is employed as the main framework for tracking and is equipped with adaptive properties to both bone contact and vein contamination. The proposed tracking method is evaluated by the experiments on both synthetic and actual data. A variety of vessels were synthesized to assess the sensitivity to the axis curvature change, obscure boundaries, and noise. The experimental results showed that the proposed method is also insensitive to parameter settings and requires less user intervention than the conventional vessel tracking methods, which proves its improved robustness.

1. Introduction

Well-timed prognosis of various diseases on cerebral arteries such as a subarachnoid hemorrhage (SAH) necessitates cerebrovascular system to be segmented out. However, cerebral arteries build up a very complex structure and pass through the cranial bone structure. Therefore, they are more difficult to extract than any other vessel structures such as cardiovascular, pulmonary vascular or hepatic vascular system in a CTA volume.

There have been a number of literatures on vessel segmentation as surveyed in [1]. The most basic method is the combination of thresholding and region-growing [2]. However, it has limitations when applied to cerebral arteries in a CTA volume due to two complications; the overlapping Hounsfield unit (HU) value distributions of bone and vessels and the close contact between them. To deal with these problems, the proposed method takes into account anatomic priors that arteries are smoothly varying structures with elliptical cross-sections. The assumption of smooth variation along the axis suggests the tracking-based segmentation as a solution. Since tracking-based approaches apply local operators on a focus inside the vessel, they can be more robust than other approaches to detect objects or features in the whole image. Furthermore, tracking methods easily allow quantitative measurements of the vessel parameters. On the other hand, tracking needs initial conditions and requires re-initialization if it gets astray or makes a turnover.

Wink et al. [3] extracted the abdominal aorta based on tracking of the center line. The abdominal aorta is thick and

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0169-2607/$ – see front matter
doi:10.1016/j.cmpb.2006.09.001

mostly straight, so it is much simpler to extract than cerebral arteries. Therefore, when applied to cerebral arteries in the CTA, their work has limitations, especially when the boundaries are obscured by vein contamination.

Shim et al. [4] have partitioned a CTA volume into the lower and upper sub-volumes and applied a separate algorithm to each one, *i.e.*, adaptive tracking method to the lower one and thresholding-based region growing to the upper one. As a consequence, it has method-inconsistency in the whole volume and the problem of global thresholds in the upper sub-volume.

The work of Florin et al. [5] is the most recent and the most related to our work. They applied the particle filter (PF) to track an elliptical cross-section for segmentation of coronaries. Their measurement model used the assumption that coronary arteries are brighter than the background, which cannot be held true for cerebral arteries of the CTA because of bone structure. Furthermore, the distance between the prediction and the actual observation does not take into account the normal vector of the cross-section but only the intensity distribution and the mean difference, which may often disturb the tracking.

The proposed work also employs the PF which is the state-of-the-art in video tracking, and defines the system and measurement models adaptively to cerebral arteries. Especially in the update stage, the border points on the cross-section perpendicular to the axis are detected as the observations $\mathbf{z}_k$. The thresholds for their detection are chosen properly according to the surroundings of the arteries. In the while, the weight associated with each particle is updated by the exponential sum of distances of the points in $\mathbf{z}_k$ and the 3D ellipse represented by each particle. In this way, the proposed tracking method updates the normal vector of the cross-section at every discrete time and lowers the chance of getting astray or making a turnover.

2. Description of the method

First of all, some basic notions on particle filtering are briefly reviewed. The detailed description is available in [7]. The PF is a sequential Monte Carlo method to solve the non-linear Bayesian state estimation problem, which can be expressed in terms of the system model and the measurement model as

$$\mathbf{x}_k = \mathbf{f}_k(\mathbf{x}_{k-1}, \mathbf{v}_{k-1}), \tag{1}$$

$$\mathbf{z}_k = \mathbf{h}_k(\mathbf{x}_k, \mathbf{w}_k) \tag{2}$$

where $\mathbf{x}_k$ and $\mathbf{z}_k$ represent the state vector and the measurement vector, respectively. Both of the system noise $\mathbf{v}_{k-1}$ and the measurement noise $\mathbf{w}_k$ are zero-mean independent identically distributed (iid) sequences. The objective of the PF is to estimate the current state $\mathbf{x}_k$ from the measurements $\mathbf{z}_{1:k} = \{\mathbf{z}_i, i = 1, \ldots, k\}$, *i.e.*, to construct the probability density function (pdf) $p(\mathbf{x}_k|\mathbf{z}_{1:k})$. With the assumption of the first-order Markov random process, the PF produces at each time k, a set of N_s particles $\{\mathbf{x}_k^i, i = 1, \ldots, N_s\}$ with a set of associated weights $\{q_k^i, i = 1, \ldots, N_s\}$ which closely approximates the posterior pdf:

$$p(\mathbf{x}_k|\mathbf{z}_k) \approx \sum_{i=1}^{N_s} q_k^i \delta(\mathbf{x}_k - \mathbf{x}_k^i). \tag{3}$$

Using the principle of importance sampling [6] and resampling at every k [7], the proposed method employs the SIR (sampling importance resampling) PF [8] and is summarized as the pseudo code in Algorithm 1. For each particle $\mathbf{x}_k^i$, the pdf $p(\mathbf{x}_k|\mathbf{z}_k)$ is obtained by the two stages of prediction and update. Then the SIR PF estimates the current state as (4) and applies the resampling stage to the whole set of the particles.

Algorithm 1. The SIR PF at the discrete time k

- FOR $i = 1 : N_s$
  - Prediction : Draw $\mathbf{x}_k^i \sim p(\mathbf{x}_k|\mathbf{x}_{k-1}^i)$
  - Update : $q_k^i \propto q_{k-1}^i p(\mathbf{z}_k|\mathbf{x}_k^i)$
- END FOR
- Estimation:

$$\hat{\mathbf{x}}_k = \text{mean}\left(\sum_{i=1}^{N_s} q_k^i \delta(\mathbf{x}_k - \mathbf{x}_k^i)\right) \tag{4}$$

- Resampling : the same way as Algorithm 2 in [7]

2.1. *The system model of the proposed method*

At the current time k, the elliptical cross-section of a vessel segment can be represented by a 9×1 state vector which is divided into three 3×1 vectors as

$$\begin{aligned}\mathbf{x}_k &= [\,c_{x_k} \quad c_{y_k} \quad c_{z_k} \quad n_{x_k} \quad n_{y_k} \quad n_{z_k} \quad a_k \quad b_k \quad \beta_k\,]^{\mathrm{T}} \\ &= [\,\mathbf{c}_k^{\mathrm{T}} \quad \mathbf{n}_k^{\mathrm{T}} \quad \mathbf{e}_k^{\mathrm{T}}\,]^{\mathrm{T}}.\end{aligned} \tag{5}$$

$\mathbf{c}_k = [\,c_{x_k} \quad c_{y_k} \quad c_{z_k}\,]^{\mathrm{T}}$ and $\mathbf{n}_k = [\,n_{x_k} \quad n_{y_k} \quad n_{z_k}\,]^{\mathrm{T}}$ represent the 3D coordinates of the ellipse center and the unit vector perpendicular to the ellipse, respectively. $\mathbf{e}_k = [\,a_k \quad b_k \quad \beta_k\,]^{\mathrm{T}}$ specifies the elliptical shape with the two lengths of the semi-axes, and the rotation angle in the normal plane represented by $\mathbf{n}_k$.

If the elliptical cross-section marches with a constant unit velocity, the state transition can be modelled as the addition of the unit normal vector to the center of the previous state. The transition is corrupted by the system noise $\mathbf{v}_k$ which can be split into three independent noise vectors of $\mathbf{v}_{\mathbf{c}_k}$, $\mathbf{v}_{\mathbf{n}_k}$, and $\mathbf{v}_{\mathbf{e}_k}$. Then the system transition of (1) is represented by three 3×1 vector equations as

$$\mathbf{c}_k = \mathbf{c}_{k-1} + \mathbf{n}_{k-1} + \mathbf{v}_{\mathbf{c}_k}, \tag{6}$$

$$\mathbf{n}_k = \mathbf{f}_{\mathbf{n}_k}(\mathbf{n}_{k-1}, \mathbf{v}_{\mathbf{n}_k}), \tag{7}$$

$$\mathbf{e}_k = \mathbf{e}_{k-1} + \mathbf{v}_{\mathbf{e}_k}. \tag{8}$$

The transitions for $\mathbf{c}_k$ and $\mathbf{e}_k$ of (6) and (8) are straightforward. All the six components of the noises, $\mathbf{v}_{\mathbf{c}_k} = [\,v_{\mathbf{c}_{x_k}} \quad v_{\mathbf{c}_{y_k}} \quad v_{\mathbf{c}_{z_k}}\,]^{\mathrm{T}}$ and $\mathbf{v}_{\mathbf{e}_k} = [\,v_{\mathbf{e}_{a_k}}\ v_{\mathbf{e}_{b_k}}\ v_{\mathbf{e}_{\beta_k}}\,]^{\mathrm{T}}$, are assumed to be independent of each other and also independent of the time index k. Then, each of them is modelled as a Gaussian distribution with zero mean and constant standard deviation denoted by $\sigma_{\mathbf{c}_x}, \sigma_{\mathbf{c}_y}, \sigma_{\mathbf{c}_z}, \sigma_{\mathbf{e}_a}, \sigma_{\mathbf{e}_b}$, and $\sigma_{\mathbf{e}_\beta}$, respectively.

For the transition of the normal vector $\mathbf{n}_k$ corrupted by the random noise $\mathbf{v}_{\mathbf{n}_k}$ as (7), more sophisticated consideration is