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On the Galerkin and collocation methods for two-point boundary value problems using sinc bases

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Abstract

This paper presents a study of the performance of the collocation and Galerkin methods using sinc basis functions for solving linear and nonlinear second-order two-point boundary value problems. The two methods have the linear systems solved by the Q-R method and have the nonlinear systems solved by Newton's method. This study shows that the collocation method performs better than the Galerkin method for the cases considered.

Keywords: Sinc function; Collocation; Galerkin; Singular; Nonlinear

1. Introduction

There is a vast amount of literature on numerical solutions of boundary-value problems involving ordinary and partial differential equations. Some of the well-known techniques used in solving these problems are finite differences, finite elements, and multi-grid methods. Aside from these classical approaches there is another important class of numerical schemes, called function space approximation methods, that includes the Rayleigh–Ritz, Galerkin and collocation methods [1].

Recently increased attention has turned to comparing numerical methods for solving two-point boundary-value problems. We consider two promising types, collocation and Galerkin methods. There are a number of hybrids of the Galerkin and collocation methods that use different types of test functions such as Cubic, Quintic, Septic splines or any polynomials and wavelets [2–5]. In the last decade, sinc bases have been used in many applications, including numerical solution of ordinary and partial differential equations. In the sinc method, the test functions are translates of the sinc function, $S(x) = \sin(\pi x)/(\pi x)$. The sinc method, which was developed by F. Stenger more than twenty years ago [6], is based on the Whittaker–Shannon–Kotel'nikov sampling theorem for entire functions. This method, which uses entire functions as bases, has many advantages over classical methods that use polynomials as bases. For example, in the presence of singularities, it gives a much better rate of convergence and greater accuracy than polynomial methods.

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The aim of this paper is to compare the sinc–Galerkin and the sinc-collocation methods for solving the two-point second-order problem

$$\mu_2(x)\, y''(x) + \mu_1(x)\, y'(x) + \mu_0(x)\, y(x) = f(x, y) \quad a < x < b \tag{1.1}$$

$$y(a) = y(b) = 0, \tag{1.2}$$

where $\mu_2(x)$, $\mu_1(x)$, $\mu_0(x)$, $f(x, y)$ and $y(x)$, are analytic functions. A comparison for the singular two-point boundary value problem

$$(x^\sigma\, y')' = f(x, y), \quad 0 < x < 1 \tag{1.3}$$

$$y(0) = A, \qquad y(1) = B, \tag{1.4}$$

where $\sigma \in (0, 1)$ and A, B are finite constants, is also made.

Numerical examples including regular, singular as well as singularly perturbed problems are considered. On the basis of these examples, it will be shown that although the sinc–Galerkin method is more popular, the sinc-collocation method gives slightly better results for the tested examples.

The paper is organized into five sections. Section 2 contains notation, definitions and some results of sinc function theory. In Section 3, the sinc–Galerkin and sinc-collocation methods are developed for linear second-order boundary value problems with homogeneous boundary conditions. The two methods are developed for nonlinear second-order boundary value problems in Section 4. Some numerical examples are presented in Section 5. Finally, Section 6 provides the conclusions of the study.

2. Preliminaries and fundamentals

The goal of this section is to recall notations and definitions of the sinc function, state some known results, and derive useful formulas that are important for this paper. First denote the set of all integers, the set of all real numbers, and the set of all complex numbers by **Z**, **R**, and **C**, respectively.

- $\mathrm{sinc}(z) = \sin(\pi z)/(\pi z),\ z \in \mathbf{C}$
  Note that $|\mathrm{sinc}(x)| \le 1$ for any $x \in \mathbf{R}$.
- $S(k, h)(z) = \mathrm{sinc}\,[(z - kh)/h]\,,\ z \in \mathbf{C},\ h > 0$
- $C(f, h)(x) = \sum_{k=-\infty}^{\infty} f(hk)\, S(k, h)(x), h > 0$
  Here, $C(f, h)(x)$ is called the Whittaker cardinal expansion of $f(x)$ whenever this series converges.
- $C_N(f, h)(x) = \sum_{k=-N}^{N} f(kh)\, S(k, h)(x)$.

The properties of Whittaker cardinal expansions have been studied and are thoroughly surveyed in [6–8]. These properties are derived in the infinite strip D_d of the complex plane where for $d > 0$

$$D_d = \left\{ \zeta = \xi + i\eta : |\eta| < d \le \frac{\pi}{2} \right\}. \tag{2.1}$$

In addition, we choose

$$h = \sqrt{\frac{\pi\, d}{\alpha\, N}}, \quad 0 < \alpha \le 1.$$

Approximations can be constructed for infinite, semi-finite, and finite intervals. To construct approximations on the interval (a, b), which are used in this paper, consider the conformal map

$$\phi(x) = \ln\left(\frac{x - a}{b - x}\right). \tag{2.2}$$

The map ϕ carries the eye-shaped region

$$D_E = \left\{ z = x + iy : \left|\arg\left(\frac{z - a}{b - z}\right)\right| < d \le \frac{\pi}{2} \right\}, \tag{2.3}$$