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Energetic balance for the flow of a second-grade fluid due to a plate subject to a shear stress

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Abstract

Exact and approximative expressions for dissipation, the power due to the shear stress at the wall and the boundary layer thickness corresponding to the unsteady motion of a second-grade fluid, induced by an infinite plate subject to a shear stress, are established. For $\alpha_1 \longrightarrow 0$, similar results for Newtonian fluids performing the same motion are obtained. The results that have been obtained here are different to those corresponding to the Rayleigh–Stokes problem. A series solution for the velocity field is also determined. Its form, as was to be expected, is identical to that resulting from the general solution using asymptotic approximations.

Keywords: Second-grade fluid; Energetic balance; Dissipation; Kinetic energy; Power

1. Introduction

The flow of non-Newtonian fluids has gained special attention due to their applications in various branches of science, engineering and technology. It is difficult to suggest a single model which exhibits all properties of these fluids, they being classified as: (1) fluids for which the shear stress depends on the shear rate; (2) fluids for which the relation between the shear stress and shear rate depends on time; and (3) fluids which possess both elastic and viscous properties, called viscoelastic fluids or elastico-viscous fluids. Although many constitutive equations have been suggested, many questions are still unresolved. Some of the continuum models do not give satisfactory results in accordance with the available experimental data. Therefore, in many practical applications, some empirical or semi-empirical equations have been used.

A constitutive equation is a relation between stress and local properties of the fluid. For a fluid at rest the stress is wholly determined by the hydrostatic pressure. In the case of a fluid in motion the relation between stress and the local properties of the fluid is more complicated. One of the most popular models for non-Newtonian fluids is the model corresponding to second-grade fluids [1]. Although there are some criticisms of the applications of this model [2–4], many papers on this have been published in the last few years [5–15]. Furthermore, it has been shown

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by Walters [16] that for many types of problems in which the flow is slow enough in the viscoelastic sense, the results given by Oldroyd's constitutive equations will be substantially similar to those from the second-grade or third-order Rivlin–Ericksen constitutive equations. Consequently, it would seem reasonable to use the second-grade or third-order constitutive equations in carrying out the calculations. This is particularly so in view of the fact that the calculation is generally simpler.

The purpose of this paper is to provide readers with an energetic study of the unsteady flow induced by an infinite flat plate that applies a specified stress in a second-grade fluid. Of special interest is the energetic balance of the three terms: changing of the kinetic energy with time, dissipation and power due to the shear stress at the wall. The last term describes the energy input that is necessary to keep the medium running. A decisive question is that of whether this term is larger or smaller than in the Newtonian case. In order to realize a good comparison between the two models, Newtonian and non-Newtonian, both exact and approximative expressions are determined. Our study is also extended to the boundary layer thickness. Its approximative expression is used to determine a series solution for the velocity field. This series solution, as was to be expected, is identical to that obtained by means of the asymptotic approximations.

2. Statement of the problem

The Cauchy stress $\mathbf{T}$ for incompressible second-grade fluids is related to the fluid motion by the constitutive equation [1–16]

$$\mathbf{T} = -p\mathbf{I} + \mu\mathbf{A}_1 + \alpha_1\mathbf{A}_2 + \alpha_2\mathbf{A}_1^2, \tag{1}$$

where $-p\mathbf{I}$ denotes the indeterminate spherical stress due to the constraint of incompressibility, μ is the dynamic viscosity, α_1 and α_2 are material moduli and $\mathbf{A}_1$ and $\mathbf{A}_2$ are the first two Rivlin–Ericksen tensors. The Clausius–Duhem inequality and the assumption that the Helmholtz free energy is minimum in equilibrium provide the following restrictions [17]:

$$\mu \geq 0, \qquad \alpha_1 \geq 0 \quad \text{and} \quad \alpha_1 + \alpha_2 = 0. \tag{2}$$

A comprehensive discussion on the restrictions for μ, α_1 and α_2 can be found in the extensive work by Dunn and Rajagopal [18]. The experimentalists have not confirmed these restrictions. The conclusion is that the fluids that have been tested are not fluids of second grade and they are characterized by a different constitutive structure.

In this paper, the unsteady flow induced by an infinite plate, that applies a shear stress in a second-grade fluid, is considered. Initially, the fluid lying over the infinite plate, that is situated in the (x, z)-plane, is at rest. At time zero, a shear stress is applied to the plate and the fluid is gradually moved. The governing equation corresponding to this motion, in the absence of a pressure gradient in the flow direction, is [1,6,7,13,14,19,20]

$$(\nu + \alpha\partial_t)\partial_y^2 u(y,t) = \partial_t u(y,t); \quad y, t > 0, \tag{3}$$

where $u(y,t)$ is the velocity, $\nu = \mu/\rho$ (ρ being the constant density of the fluid) is the kinematic viscosity of the fluid and $\alpha = \alpha_1/\rho$. For these motions the constraint of incompressibility is automatically satisfied and the balance of the linear momentum leads to the meaningful equation

$$\partial_y \tau(y,t) = \rho\partial_t u(y,t), \tag{4}$$

where $\tau(y,t) = S_{xy}(y,t)$ is the shear stress.

The appropriate initial and boundary conditions, resulting from [19,20], are

$$u(y,0) = 0; \quad y > 0, \tag{5}$$

$$(\nu + \alpha\partial_t)\partial_y u(y,t) = \frac{\tau(y,t)}{\rho} = \frac{f}{\rho} \quad \text{at } y = 0,\ t > 0, \tag{6}$$

$$u(y,t),\ \partial_y u(y,t) \longrightarrow 0 \quad \text{for } y \longrightarrow \infty,\ t > 0, \tag{7}$$

where the constant f is the shear stress applied to the plate.