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Exact solutions of discrete complex cubic Ginzburg–Landau equation via extended tanh-function approach

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Abstract

In this paper, we obtained rich solutions for the discrete complex cubic Ginzburg–Landau equation by means of the extended tanh-function approach. These solutions include chirpless bright soliton, chirpless dark soliton, triangular function solutions and some solutions with alternating phases, and so on. Meanwhile, the range of parameters where some exact solution exists is given.

Keywords: Extended tanh-function approach; Nonlinear differential–difference equations; Discrete complex cubic Ginzburg–Landau equation; Exact solutions; Range of parameters

1. Introduction

Discrete solitons in nonlinear lattices have been the focus of considerable attention in diverse branches of science [1]. Discrete solitons have been demonstrated to exist in a wide range of physical systems, e.g. atomic chains [2] (discrete lattices) with on-site cubic nonlinearities, molecular crystals [3], biophysical systems [4], electrical lattices [5] and Bose–Einstein condensates [6]. Recently, the existence of discrete solitons in photonic structures (in arrays of coupled nonlinear optical wave guides [7] and in a nonlinear photonic crystal structure [8]) was announced and has attracted considerable attention in the scientific community. Photonic crystals, which are artificial microstructures having photonic bandgaps, can be used to precisely control the propagation of optical pulses and beams. Furthermore, Ablowitz et al. [9] developed a fully discrete perturbation theory and show that slowly moving discrete solitons are "chirp". When using discrete waveguides and photonic crystals, "discrete solitons" appear naturally and have a number of interesting properties. Many scientists believe that the discrete solitons can have an important role in this technology.

Discrete Ginzburg–Landau (DGL) models have also been considered in the literature [10–12]. These DGL lattices are quite often used to describe a number of physical systems such as Taylor and frustrated vortices in hydrodynamics [10] and semiconductor laser arrays in optics [11]. In these latter studies, the DGL model has been predominantly used in connection with spatiotemporal chaos, instabilities, and turbulence [12]. Most studies related to

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discrete solitons are directed at conservative systems, i.e., those that preserve energy. However, dissipative systems are more common in nature, so further studies on discrete dissipative systems are certainly required. Ravoux et al. [13] studied the discrete analog of the complex cubic Ginzburg–Landau equation having pattern formation phenomena in mind. In particular, they studied plane wave instability in such systems. Soto-Crespo et al. [14] studied the discrete complex cubic Ginzburg–Landau (DCCGL) equation having several exact solutions. We consider a discrete equation set (a model of a dissipative system), viz., the following discrete complex cubic Ginzburg–Landau (DCCGL) equation [15]

$$\mathrm{i}\frac{\mathrm{d}\psi_n}{\mathrm{d}t}+\left(\frac{D}{2}-\mathrm{i}\beta\right)(\psi_{n+1}-2\psi_n+\psi_{n-1})+(1-\mathrm{i}\epsilon)|\psi_n|^2(\psi_{n+1}+\psi_{n-1})-\mathrm{i}\gamma\psi_n=0, \tag{1}$$

where ψ_n is complex variable defined for all integer values of the site index n. $\psi_{n+1}-2\psi_n+\psi_{n-1}$ plainly approximates a second derivative term for a continuous system, and thus D is the coefficient of the diffraction term. γ, ϵ, β are the linear dissipation, cubic nonlinear amplification and filter coefficients, respectively. In the limit of $\beta = \epsilon = \gamma = 0$, Eq. (1) is reduced to the integrable discrete nonlinear Schrodinger equation (AL model) [16]. The continuous limit of Eq. (1) is the complex Ginzburg–Landau equation (CGLE) [17]

$$\mathrm{i}\frac{\mathrm{d}\psi}{\mathrm{d}t}+\left(\frac{D}{2}-\mathrm{i}\beta\right)\psi_{xx}+(1-\mathrm{i}\epsilon)|\psi|^2\psi-\mathrm{i}\gamma\psi=0, \tag{2}$$

which has many applications in describing non-equilibrium systems, phase transitions, and wave propagation phenomena. When $\beta = \epsilon = \gamma = 0$, Eq. (2) is reduced to the NLS equation.

With the development of symbolic computation, many direct and effective methods are presented to solve nonlinear differential–difference equations (NDDEs). For instance, Baldwin et al. [18] derived the kink-type solutions of many spatially discrete nonlinear models in terms of tanh function. Recently, Dai et al. [19] obtained the kink-type solutions of the discrete sine-Gordon equation by means of the hyperbolic function approach. More recently, the Jacobian elliptic function method is generalized to solve differential–difference equations [20]. Moreover, the solutions of the integrable discrete nonlinear Schrödinger equation (AL model) are derived using the extended Jacobian elliptic function method [21]. However, these methods [19–21] with much complicated calculations cannot give us an unified formulation to construct exact solutions. Thus one is devoted to finding the suitable and simple methods, which are extensively and successfully applied in many nonlinear partial differential equations to obtain exact solutions in a uniform way, to solve differential–difference equations. Nevertheless, these methods are hardly generalized to solve differential–difference equations because of the difficulty to search iterative relations between lattice indices, for example, the relations from indices n to $n \pm 1$. Fortunately, by careful analysis, we present the extended tanh-function method for differential–difference equations and successfully find the iterative relations between lattice indices. The virtue of this proposed method is that, without much complicated calculations, we circumvent integration to directly get many exact solutions in a uniform way. Another feature of this method is that it provides us a guideline to classify the various types of the solution according to the parameter δ [the meaning of δ see (6) and (7) in Section 2]. Applying this method, we investigate the discrete complex cubic Ginzburg–Landau (DCCGL) equation (1) and obtain chirpless bright soliton, chirpless dark soliton, triangular function solutions and some solutions with alternating phases.

2. Extended tanh-function method for NDDEs

In this section, we would like to outline the extended tanh-function method for NDDEs step-by-step.

Consider a system of M polynomial DDEs

$$\triangle(\mathbf{u_{n+p_1}}(\mathbf{x}),\ldots,\mathbf{u_{n+p_k}}(x),\ldots,\mathbf{u'_{n+p_1}}(\mathbf{x}),\ldots,\mathbf{u'_{n+p_k}}(\mathbf{x}),\ldots,\mathbf{u^{(r)}_{n+p_1}}(\mathbf{x}),\ldots,\mathbf{u^{(r)}_{n+p_k}}(\mathbf{x}))=0, \tag{3}$$

where the dependent variable $\mathbf{u_n}$ has M components $u_{i,n}$, the continuous variable $\mathbf{x}$ has N components x_i, the discrete variable $\mathbf{n}$ has Q components n_j, the k shift vectors $\mathbf{p}_i$, and $\mathbf{u^{(r)}}(\mathbf{x})$ denotes the collection of mixed derivative terms of order r.

According to the tanh-function method [18,19], the main steps of the extended tanh-function method for NDDEs are outlined as follows.