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On the approximation problem of common fixed points for a finite family of non-self asymptotically quasi-nonexpansive-type mappings in Banach spaces

Y.X. Tian[a], S.S. Chang[b,*], J.L. Huang[b]

[a] *College of Computers, Chongqing Post Telecommunications University, Chongqing, 400065, China*
[b] *Department of Mathematics, Yibin University, Yibin, Sichuan 644007, China*

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Abstract

The purpose of this paper is to introduce the concept of non-self asymptotically quasi-nonexpansive-type mappings and to construct a iterative sequence to converge to a common fixed point for a finite family of non-self asymptotically quasi-nonexpansive-type mappings in Banach spaces. The results presented in this paper improve and extend the corresponding results in Alber, Chidume and Zegeye [Ya.I. Alber, C.E. Chidume, H. Zegeye, Approximating of total asymptotically nonexpansive mappings, Fixed Point Theory and Applications (2006) 1–20. Article ID10673], Ghosh and Debnath [M.K. Ghosh, L. Debnath, Convergence of Ishikawa iterates of quasi-nonexpansive mappings, Journal of Mathematical Analysis and Applications 207 (1997) 96–103], Liu [Q.H. Liu, Iterative sequences for asymptotically quasi-nonexpansive type mappings, Journal of Mathematical Analysis and Applications 259 (2001) 1–37; Q.H. Liu, Iterative sequences for asymptotically quasi-nonexpansive mappings with error member, Journal of Mathematical Analysis and Applications 259 (2001) 18–24; Q.H. Liu, Iteration sequences for asymptotically quasi-nonexpansive mapping with an error member of uniform convex Banach space, Journal of Mathematical Analysis and Applications 266 (2002) 468–471], Petryshyn [W.V. Petryshyn, T.E. Williamson Jr., Strong and weak convergence of the sequence of successive approximations for quasi-nonexpansive mappings, Journal of Mathematical Analysis and Applications 43 (1973) 459–497], Quan and Chang [J. Quan, S.S. Chang, X.J. Long, Approximation common fixed point of asymptotically quasi-nonexpansive type mappings by the finite steps iterative sequences, Fixed Point Theory and Applications V (2006) 1–38. Article ID 70830], Shahzad and Udomene [N. Shahzad, A. Udomene, Approximating common fixed point of two asymptotically quasi-nonexpansive mappings in Banach spaces, Fixed Point Theory and Applications (2006) 1–10. Article ID 18909] Xu [B.L. Xu, M.A. Noor, Fixed-point iterations for asymptotically nonexpansive mappings in Banach spaces, Journal of Mathematical Analysis and Applications 267 (2002) 444–453], Zhang [S.S. Zhang, Iterative approximation problem of fixed points for asymptotically nonexpansive mappings in Banach spaces, Acta Mathematicae Applicatae Sinica 24 (2001) 236–241] and Zhou and Chang [Y.Y. Zhou, S.S. Chang, Convergence of implicit iteration process for a finite family of asymptotically nonexpansive mappings in Banach spaces, Numerical Functional Analysis and Optimization 23 (2002) 911–921].

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* Corresponding author. Tel.: +86 28 85415930.
E-mail addresses: tianyx@cqupt.edu.cn (Y.X. Tian), sszhang_1@yahoo.com.cn (S.S. Chang), jialinh2880@163.com (J.L. Huang).

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1. Introduction and preliminaries

Throughout this paper, we assume that E is a real Banach space, C is a nonempty closed convex subset of E and $F(T)$ is the set of fixed points of mapping T.

Definition 1.1. Let $T : C \to C$ be a mapping.

(1) T is said to be *nonexpansive*, if $\|Tx - Ty\| \leq \|x - y\|$ for every $x, y \in C$;
(2) T is said to be *asymptotically nonexpansive* [1,12] if there exists a sequence $\{k_n\} \subset [1, \infty)$ with $k_n \to 1$ as $n \to \infty$ such that

$$\|T^n x - T^n y\| \leq k_n \|x - y\|, \quad \forall x, y \in C;\ n \geq 1.$$

Definition 1.2. Let E be a real Banach space and C be a nonempty subset of E.

(1) A mapping P from E onto C is said to be a *retraction*, if $P^2 = P$;
(2) If there exists a continuous retraction $P : E \to C$ such that $Px = x, \forall x \in C$, then the set C is said to be a *retract of* E.
(3) In particular, if there exists a nonexpansive retraction $P : E \to C$ such that $Px = x, \forall x \in C$, then the set C is said to be a *nonexpansive retract of* E.

Next we introduce the following concepts for non-self mapping:

Definition 1.3. Let E be a real Banach space, C be a nonempty nonexpansive retract of E and P be the nonexpansive retraction from E onto C. Let $T : C \to E$ be a non-self mapping.

(1) T is said to be a *non-self asymptotically nonexpansive mapping* [4], if there exists a sequence $\{k_n\} \subset [1, \infty)$ with $\lim_{n\to\infty} k_n = 1$ such that

$$\|T(PT)^{n-1}x - T(PT)^{n-1}y\| \leq k_n \|x - y\| \quad \forall x, y \in C,\ n \geq 1$$

(2) T is said to be a *non-self asymptotically quasi-nonexpansive mapping*, if $F(T) \neq \emptyset$ and there exists a sequence $\{k_n\} \subset [1, \infty)$ with $\lim_{n\to\infty} k_n = 1$ such that

$$\|T(PT)^{n-1}x - p\| \leq k_n \|x - p\| \quad \forall x \in C, p \in F(T)\ n \geq 1;$$

(3) T is said to be a *non-self asymptotically nonexpansive-type mapping*, if

$$\limsup_{n\to\infty} \{ \sup_{x,y\in C} [\|T(PT)^{n-1}x - T(PT)^{n-1}y\| - \|x - y\|]\} \leq 0;$$

(4) T is said to be a *non-self asymptotically quasi-nonexpansive-type mapping*, if $F(T) \neq \emptyset$ and

$$\limsup_{n\to\infty} \{\sup_{x\in C} [\|T(PT)^{n-1}x - p\| - \|x - p\|]\} \leq 0 \quad \forall p \in F(T).$$

Remark. It follows from Definition 1.3 that

(a) if $T : C \to E$ is a non-self asymptotically nonexpansive mapping, then T is a non-self asymptotically nonexpansive-type mapping;
(b) if $T : C \to E$ is a non-self asymptotically quasi-nonexpansive mapping, then T is a non-self asymptotically quasi-nonexpansive-type mapping
(c) If $F(T)$ is nonempty and T is non-self asymptotically nonexpansive-type mapping, then T is a non-self asymptotically quasi-nonexpansive-type mapping.

Definition 1.4. Let E be a real Banach space and C be a nonempty closed convex subset of E which is also a nonexpansive retract of E with a retraction P. Let $T_1, T_2, \ldots, T_N : C \to E$ be non-self asymptotically quasi-nonexpansive-type mappings. Let $x_1 \in C$ be any given point. Then the sequence $\{x_n\}$ defined by