Available online at www.sciencedirect.com
ScienceDirect
Computers and Mathematics with Applications 53 (2007) 1867–1895
An International Journal
computers & mathematics with applications
www.elsevier.com/locate/camwa

A finite element method for investigating general elastic multi-structures

Jianguo Huang[a,*,1], Ling Guo[a], Zhongci Shi[b]

[a] Department of Mathematics, Shanghai Jiao Tong University, Shanghai 200240, PR China
[b] Institute of Computational Mathematics, Academy of Mathematics and System Sciences, Chinese Academy of Sciences, Beijing 100080, PR China

Received 26 July 2006; accepted 4 August 2006

Abstract

A finite element method is proposed for investigating the general elastic multi-structure problem, where displacements on bodies, longitudinal displacements on plates, longitudinal displacements and rotational angles on rods are discretized using conforming linear elements, transverse displacements on plates and rods are discretized respectively using TRUNC elements and Hermite elements of third order, and the discrete generalized displacement fields in individual elastic members are coupled together by some feasible interface conditions. The unique solvability of the method is verified by the Lax–Milgram lemma after deriving generalized Korn's inequalities in some nonconforming element spaces on elastic multi-structures. The quasi-optimal error estimate in the energy norm is also established. Some numerical results are presented at the end.

Keywords: Elastic multi-structures; Finite elements; Generalized Korn's inequality; Unique solvability; Error estimates

1. Introduction

Elastic multi-structures are usually assembled from elastic substructures of the same or different dimensions (bodies, plates, rods, etc.) with proper junctions, which are widely encountered in engineering applications. In the past few decades, many researchers have been interested in mathematical modeling and numerical solutions for simple elastic multi-structures composed of only two elastic members [1–10]. However, there are few considerations about the general elastic multi-structure problem. Feng and Shi [11,12] established mathematical models for general elastic multi-structures via the variational principle, after reasonable presentation for the interface conditions among substructures. The corresponding mathematical theory was developed in [13] by Huang, Shi, and Xu. In this paper, we plan to propose and analyze a finite element method for investigating the general elastic multi-structure problem. We mention the following words of Ciarlet to show the importance of such studies: "A challenging program consists in

* Corresponding author.
E-mail addresses: jghuang@sjtu.edu.cn (J. Huang), citycolor@sjtu.edu.cn (L. Guo), shi@lsec.cc.ac.cn (Z. Shi).
[1] At present also engaged by Division of Computational Science, E-Institute of Shanghai Universities, Shanghai Normal University, PR China.

0898-1221/$ - see front matter
doi:10.1016/j.camwa.2006.08.048

numerically approximating the mathematical models of elastic multi-structures that comprise many substructures.” [4, p. 180].

Let there be given N_3 body members $\Omega^3 := \{\alpha_1, \dots, \alpha_{N_3}\}$, N_2 plate members $\Omega^2 := \{\beta_1, \dots, \beta_{N_2}\}$, and N_1 rod (beam) members $\Omega^1 := \{\gamma_1, \dots, \gamma_{N_1}\}$. They are rigidly connected to form an elastic multi-structure [12,13]:

$$\Omega = \{\alpha_1, \dots, \alpha_{N_3}; \beta_1, \dots, \beta_{N_2}; \gamma_1, \dots, \gamma_{N_1}\}.$$

Assume that Ω fulfills the following four conditions:

1. Each body member α is a bounded polyhedron and each plate member β is a bounded polygon.
2. Ω is geometrically connected in the sense that for any two points in Ω, one can connect them by a continuous path consisting of a finite number of line segments each of which belongs to some elastic member in Ω.
3. For any two adjacent elastic members $\mathcal{A}$ and $\mathcal{B}$, the dimension of the intersection $\bar{\mathcal{A}} \cap \bar{\mathcal{B}}$ can only differ from the dimensions of these two members by one dimension at most; for example, a body member can only have body or plate members as its adjacent elastic members.
4. Ω is geometrically conforming in the sense that if $\mathcal{A}$ and $\mathcal{B}$ are two adjacent elastic members in Ω with the same dimension, then $\partial\mathcal{A} \cap \partial\mathcal{B}$ should be the common boundary of $\mathcal{A}$ and $\mathcal{B}$.

We point out that the first condition is given for ease of exposition, and the second one is satisfied generally for practical problems. But the remaining two conditions may not be satisfied for some elastic multi-structures. In this case, one can transform the original structures into new ones satisfying such conditions by adding or changing some individual elastic members; we refer the reader to [12] for details along this line.

We denote all proper boundary area elements of bodies by

$$\Gamma^2 := \{\beta_{N_2+1}, \dots, \beta_{N_2'}\} = \Gamma_1^2 \cup \Gamma_2^2,$$

where $\Gamma_1^2 := \{\beta_{N_2+1}, \dots, \beta_{N_2+M_2}\}$ and $\Gamma_2^2 := \{\beta_{N_2+M_2+1}, \dots, \beta_{N_2'}\}$. Here Γ_1^2 consists of all external proper boundary area elements while Γ_2^2 consists of all interfaces of bodies. Analogously, denote all proper boundary lines of plates by

$$\Gamma^1 := \{\gamma_{N_1+1}, \dots, \gamma_{N_1'}\} = \Gamma_1^1 \cup \Gamma_2^1,$$

where $\Gamma_1^1 := \{\gamma_{N_1+1}, \dots, \gamma_{N_1+M_1}\}$ and $\Gamma_2^1 := \{\gamma_{N_1+M_1+1}, \dots, \gamma_{N_1'}\}$. Here Γ_1^1 consists of all external boundary lines while Γ_2^1 consists of all interfaces of plates. Denote all boundary points of rods by $\Gamma^0 := \{\delta_1, \dots, \delta_{N_0}\}$, and all corner points of proper boundaries of plates by $\Gamma_3^0 := \{\delta_{N_0+1}, \dots, \delta_{N_0'}\}$ (except those in Γ^0). Let $\Gamma^0 = \Gamma_1^0 \cup \Gamma_2^0$ with

$$\Gamma_1^0 := \{\delta_1, \dots, \delta_{M_0}\}, \qquad \Gamma_2^0 := \{\delta_{M_0+1}, \dots, \delta_{N_0}\}.$$

Here Γ_1^0 consists of all external boundary points while Γ_2^0 consists of all common boundary points. An element of Ω^3, $\Omega^2 \cup \Gamma^2$, $\Omega^1 \cup \Gamma^1$, and $\Gamma^0 \cup \Gamma_3^0$ is called respectively a body, area, line, and point element.

We introduce a right-handed orthogonal system (x_1, x_2, x_3) in the space R^3, whose orthonormal basis vectors are denoted by $\{\boldsymbol{e}_i\}_{i=1}^3$. With each elastic member in Ω, we associate a local right-handed coordinate system $(x_1^\omega, x_2^\omega, x_3^\omega)$ as follows. ($\{\boldsymbol{e}_i^\omega\}_{i=1}^3$ represent the related orthonormal basis vectors.) For a body member $\alpha \in \Omega^3$, its local coordinate system is chosen as the global system (x_1, x_2, x_3), and let $\boldsymbol{n}^\alpha$ be the unit outward normal to the boundary $\partial\alpha$ of α. For a plate member $\beta \in \Omega^2$, x_1^β and x_2^β are its longitudinal directions, and x_3^β the transverse direction. Moreover, along the boundary $\partial\beta$ of β, a unit tangent vector $\boldsymbol{t}^\beta$ is selected such that $\{\boldsymbol{n}^\beta, \boldsymbol{t}^\beta, \boldsymbol{e}_3^\beta\}$ forms a right-handed coordinate system, where $\boldsymbol{n}^\beta$ denotes the unit outward normal to $\partial\beta$ in the longitudinal plane, and $\boldsymbol{e}_3^\beta$ the unit transverse vector of β. For a rod line element $\gamma \in \Omega^1$, x_1^γ is the longitudinal direction, x_2^γ and x_3^γ are the transverse directions, and the origin of the local coordinates is located at an endpoint of γ. For a line element $\gamma \in \Gamma^1$, let $\boldsymbol{e}_1^\gamma$ be a unit vector representing the longitudinal direction of γ.