Available online at www.sciencedirect.com
SCIENCE @ DIRECT®
ELSEVIER
Computers and Mathematics with Applications 52 (2006) 429–438
An International Journal
computers & mathematics with applications
www.elsevier.com/locate/camwa

Periodic Solutions for a Two-Neuron Network with Delays

BINGWEN LIU
College of Mathematics and Information Science
Jiaxing University
Jiaxing, Zhejiang 314001, P.R. China
liubw007@yahoo.com.cn

LIHONG HUANG
College of Mathematics and Econometrics
Hunan University
Changsha 410082, P.R. China

(Received September 2004; revised and accepted March 2006)

Abstract—In this paper, we use the coincidence degree theory to establish new results on the existence of ω-periodic solutions for a two-neuron network model.

Keywords—Two-neuron network model, Periodic solution, Existence, Coincidence degree theory, Delays.

1. INTRODUCTION

Consider the following model for an artificial network of two neurons,

$$\begin{aligned} x_1'(t) &= -c_1 x_1(t) + \sum_{j=1}^{2} a_{1j} f_j(x_j(t)) + \sum_{j=1}^{2} b_{1j} f_j(x_j(t-\tau_{1j})) + I_1(t), \\ x_2'(t) &= -c_2 x_2(t) + \sum_{j=1}^{2} a_{2j} f_j(x_j(t)) + \sum_{j=1}^{2} b_{2j} f_j(x_j(t-\tau_{2j})) + I_2(t), \end{aligned} \tag{1.1}$$

where $x_1(t)$ and $x_2(t)$ denote the activations of corresponding neurons, $c_i > 0$, $i = 1,2$, are the internal decay rate, $\tau_{ij} > 0$, $i,j = 1,2$, are the synaptic transmission delays, a_{ij} and b_{ij}, $i,j = 1,2$, are the synaptic weights, $f_i \in C(R,R)$, $i = 1,2$, are the activation functions, and $I_i \in C(R,R)$, $i = 1,2$, are the external inputs with periodic $\omega > 0$.

It is well known that system (1.1) describes the evolution of the so-called cellular neural network of two neurons. In recent years, the problem of the existence of periodic solutions of system (1.1)

This work was supported by the NNSF (10371034) of China, the Doctor Program Foundation of the Ministry of Education of China (20010532002) and the Key Object of Chinese Ministry of Education ([2002]78).
The authors would like to express their sincere appreciation to the reviewer for his/her helpful comments in improving the presentation and quality of this manuscript.

0898-1221/06/$ - see front matter Typeset by $\mathcal{AMS}$-TEX
doi:10.1016/j.camwa.2006.03.024

has been extensively studied in the literature. We refer the reader to [1–7] and the references cited therein. Moreover, in the above-mentioned literature, we observe the following assumptions.

(H_0) For each $j \in \{1,2\}$, $f_j : R \to R$ is Lipschitz with Lipschitz constant L_j, i.e.,

$$|f_j(u_j) - f_j(v_j)| \le L_j |u_j - v_j|, \qquad \text{for all } u_j, v_j \in R.$$

(H_1) There exist two nonnegative constants p_j and q_j such that

$$|f_j(u)| \le p_j |u| + q_j, \qquad \text{for all } u \in R, \quad j = 1,2;$$

have been considered as fundamental for the considered existence of periodic solutions of system (1.1). However, to the best of our knowledge, few authors have considered system (1.1) without the assumptions (H_0) and (H_1). Thus, it is worth while to continue to investigate the existence of periodic solutions of system (1.1).

In this paper, by using the continuation theorem of coincidence degree theory, we will give some results on the existence of the ω-periodic solution to system (1.1). The results of this paper are new and they complement previously known results. In particular, we do not need the assumptions (H_0) and (H_1). An illustrative example is given in Section 4.

2. PRELIMINARIES

First, consider an abstract equation in a Banach space X,

$$Lx = \lambda N x, \qquad \lambda \in (0,1), \tag{2.1}$$

where $L : \operatorname{Dom} L \cap X \to X$ is a linear operator and λ is a parameter. Let P and Q denote two projectors,

$$P : \operatorname{Dom} L \cap X \to \operatorname{Ker} L \text{ and } Q : X \to X/\operatorname{Im} L.$$

For convenience, we introduce a continuation theorem [8, p. 40] as follows.

LEMMA 2.1. *Let X be a Banach space. Suppose that $L : \operatorname{Dom} L \subset X \longrightarrow X$ is a Fredholm operator with index zero and $N : \bar{\Omega} \longrightarrow X$ is L-compact on $\bar{\Omega}$ with Ω open bounded in X. Moreover, assume that all the following conditions are satisfied:*

(1) $Lx \neq \lambda N x$, $\forall\, x \in \partial\Omega \cap \operatorname{Dom} L, \lambda \in (0,1)$;
(2) $QNx \neq 0$, $\forall\, x \in \partial\Omega \cap \operatorname{Ker} L$;
(3) $\deg_B\{QN, \Omega \cap \operatorname{Ker} L, 0\} \neq 0$, $\deg_B$ *denotes the Brouwer degree.*

Then, equation $Lx = Nx$ has at least one solution in $\bar{\Omega}$.

For ease of exposition, throughout this paper, we will adopt the following notations:

$$|x_i|_\infty = \max_{t\in[0,\omega]} |x_i(t)|, \quad u(t) = (x_1(t), x_2(t))^\top, \quad |x_i|_k = \left(\int_0^\omega |x_i(t)|^k \, dt\right)^{1/k}, \qquad i = 1,2.$$

We denote X as the set of all continuously ω-periodic functions $u(t)$ defined on R, and denote $\|u\|_X = \max\{|x_1|_\infty, |x_2|_\infty\}$. Then, X is a Banach space when it is endowed with the norm $\|u\|_X$. Let for $u(t) = (x_1(t), x_2(t))^\top \in X$,

$$(Nu)(t) = \begin{pmatrix} -c_1 x_1(t) + \sum_{j=1}^{2} a_{1j} f_j(x_j(t)) + \sum_{j=1}^{2} b_{1j} f_j(x_j(t-\tau_{1j})) + I_1(t) \\ -c_2 x_2(t) + \sum_{j=1}^{2} a_{2j} f_j(x_j(t)) + \sum_{j=1}^{2} b_{2j} f_j(x_j(t-\tau_{2j})) + I_2(t) \end{pmatrix}, \tag{2.2}$$