

# On the enrichment zone size for optimal convergence rate of the Generalized/Extended Finite Element Method



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## ABSTRACT

Singular enrichment functions are broadly used in Generalized or Extended Finite Element Methods (GFEM/XFEM) for linear elastic fracture mechanics problems. These functions are used at finite element nodes within an enrichment zone around the crack tip/front in 2- and 3-D problems, respectively. Small zones lead to suboptimal convergence rate of the method while large ones lead to ill-conditioning of the system of equations and to a large number of degrees of freedom. This paper presents an *a priori* estimate for the minimum size of the enrichment zone required for optimal convergence rate of the GFEM/XFEM. The estimate shows that the minimum size of the enrichment zone for optimal convergence rate depends on the element size and polynomial order of the GFEM/XFEM shape functions. Detailed numerical verification of these findings is also presented.

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## 1. Introduction

The advent of Partition of Unity methods [1,2], like the *hp*-cloud method [3,2,4], the Generalized or Extended Finite Element Method (GFEM/XFEM) [5,6,1,7–9], and the Particle Partition of Unity Method [10,11], has greatly facilitated the computation of accurate and efficient numerical solutions for problems with singularities. Of particular engineering relevance are linear elastic fracture mechanics problems with stationary or propagating cracks. The rapid growth and development of the GFEM/XFEM in the last two decades has led to a phenomenal increase in the number of users of these methods and its availability in mainstream commercial finite element software like Abaqus [12] and LS-DYNA [13]. The main idea behind the GFEM/XFEM is to incorporate *a priori* knowledge about the solution of a problem into the finite element solution space using the partition of unity property of finite element shape functions. It is to be noted that GFEM and XFEM are essentially the same methods, as discussed in [14]. The names GFEM and XFEM are used interchangeably in this paper.

Several researchers have exploited the robustness and flexibility associated with the GFEM/XFEM to solve elasticity problems involving cracks [15–20]. This method relaxes meshing constraints imposed by the standard Finite Element Method (FEM) for modeling cracks or moving interfaces. In addition, it improves the numerical accuracy while retaining the attractive features of the FEM. In problems involving cracks, the singularity is resolved poorly by the polynomial shape functions used in the FEM, unless a highly-refined mesh is used close to the crack tip. The GFEM alleviates this problem by building a solution space containing *a priori* knowledge about the elasticity solution in the neighborhood of cracks.

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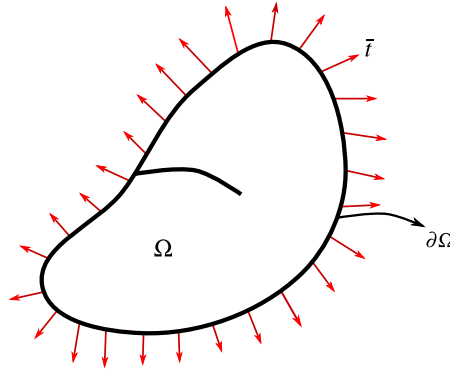


Fig. 1. Linear elastic boundary value problem with a crack in 2-D.

The GFEM can handle discontinuities and singularities independently of the finite element mesh by proper selection of local approximation spaces in pre-selected regions of the problem domain. This is accomplished through the so-called *enrichment functions*. For problems involving cracks, two types of enrichment functions are typically adopted [21,19,22]: (i) Heaviside functions able to represent the discontinuity of the elasticity solution across the crack surface and (ii) Westergaard asymptotic singular displacement fields, which approximate the singularity and discontinuity of the elasticity solution near the crack tip.

Most GFEM formulations for fractures [15,21,19] have adopted singular enrichment functions only at the nodes of elements containing the crack tip in 2-D or intersected by the crack front in 3-D. This enrichment strategy, referred to as *topological enrichment* [23,24], leads to the same suboptimal convergence behavior as the standard FEM on quasi-uniform meshes. Laborde et al. [23] and Béchet et al. [24] proposed the idea of enriching finite element nodes in a fixed neighborhood around the crack tip/front. This so-called *geometrical enrichment* strategy leads to optimal convergence rate, as in problems with smooth solutions, provided proper singular enrichment functions are adopted [25]. A brief overview of these enrichment strategies is presented in Section 4. Other researchers [26,27] have also numerically demonstrated the need for geometrical enrichment around the crack tip/front in order to obtain optimal convergence rate.

The geometrical enrichment zone with singular enrichment functions can be chosen arbitrarily large. However, large enrichment zones lead to ill-conditioned stiffness matrices as shown in [28,29] and to a larger number of degrees of freedom than the topological enrichment strategy. Therefore, estimates of the minimum size of the enrichment zone required for optimal convergence rate of the GFEM are needed. To the authors' knowledge, no guidelines for the selection of enrichment zone sizes in the GFEM/XFEM are available in the literature. This paper presents an *a priori* estimate for the minimum size of the enrichment zone. The estimate shows that the minimum size depends on the element size and polynomial order of the GFEM shape functions. Numerical verification of these findings is also presented.

After this introduction, Section 2 describes the linear elastic fracture mechanics problem considered in this study, followed by a brief review of the Generalized Finite Element Method (GFEM) in Section 3. Section 4 reviews enrichment strategies commonly adopted in the neighborhood of a crack tip. Section 4.3 presents an *a priori* estimate of the minimum size of the enrichment zone for linear elastic fracture mechanics problems. Numerical experiments aimed at the verification of the proposed estimate are presented in Section 5. Finally, Section 6 summarizes the main results and conclusions of this study.

## 2. Model problem definition

Consider a cracked domain,  $\bar{\Omega} = \Omega \cup \partial\Omega$  in  $\mathbb{R}^2$ , like the one shown in Fig. 1.

The equilibrium and constitutive equations are given by

$$\nabla \cdot \sigma = \mathbf{0} \quad \sigma = \mathbf{C} : \varepsilon \quad \text{in } \Omega \quad (1)$$

where  $\mathbf{C}$  is Hooke's tensor,  $\sigma$  denotes the Cauchy stress tensor, and  $\varepsilon$  is the small strain tensor. The following boundary conditions are prescribed on  $\partial\Omega$

$$\sigma \cdot \mathbf{n} = \bar{\mathbf{t}} \quad \text{on } \partial\Omega \quad (2)$$

where  $\mathbf{n}$  is the outward unit normal vector to  $\partial\Omega$  and  $\bar{\mathbf{t}}$  are prescribed tractions. The crack surface is assumed to be traction-free, i.e.,  $\bar{\mathbf{t}} = \mathbf{0}$  on the crack surface. Eqs. (1) and (2) are the strong form of governing equations.

The weak formulation of the problem above is given by the Principle of Virtual Work, which reads

Find  $\mathbf{u} \in \mathcal{E}(\Omega)$ , such that  $\forall \mathbf{v} \in \mathcal{E}(\Omega)$

$$B(\mathbf{u}, \mathbf{v}) = F(\mathbf{v}) \quad (3)$$

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