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Fast simulation of multi-dimensional wave problems by the sparse scheme of the method of fundamental solutions

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ABSTRACT

In this work, a meshless scheme is presented for the fast simulation of multi-dimensional wave problems. The present method is rather simple and straightforward. The Houbolt method is used to eliminate the time dependence of spatial variables. Then the original wave problem is converted into equivalent systems of modified Helmholtz equations. The sparse scheme of the method of fundamental solutions in combination with the localized method of approximate particular solutions is employed for efficient implementation of spatial variables. To demonstrate the effectiveness and simplicity of this new approach, three numerical examples have been assessed with excellent performance.

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1. Introduction

In recent years, the development of numerical solvers for the simulation of wave propagation problems has been of great importance to many branches of sciences and engineering aspects [1–4]. The wave equations govern many physical phenomena such as acoustic wave propagation [5,6], mechanical vibrations on strings [7], water wave propagation [8], etc. In this paper, a meshless numerical model which is based on the sparse scheme of the method of fundamental solutions (SMFS) [9], the localized method of approximate particular solutions (LMAPS) [10–12], and the Houbolt method [13,14] is developed for the fast simulation of two and three dimensional wave problems.

The method of fundamental solutions (MFS) [15–18] is one of the most well-known boundary-type meshless methods which approximates the solution by a linear combination of fundamental solutions with singularities placed outside the physical domain. Due to development of the fictitious boundary in the MFS, singular integrals of fundamental solutions are completely avoided. The unknown coefficients preceding the fundamental solutions are then determined in order to satisfy boundary conditions. Since the MFS was attributed to Kupradze in 1964 [19], it has become increasingly popular and has found extensive applications in a broad range of engineering problems primarily due to the efforts of Golberg and Chen who have successfully extended the MFS to non-homogeneous problems and various types of time-dependent problems by introducing the method of particular solutions (MPS) [20,21]. However, there are several inherent drawbacks concerning the

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MFS which have not yet been satisfactorily addressed. The first fundamental question is the location of the fictitious source points. One strategy concerning this problem is to let the coordinates of sources be a part of the solution which will result in a nonlinear least-squares problem. However, this approach could be costly expensive and will lead to a nonlinear problem even for linear problems. Alternatively, the locations of the sources can be pre-assigned on the pseudo-boundary. The second drawback is that it is difficult to solve large scale problems with MFS. After discretization of the given problem, the MFS produces an ill-conditioned dense matrix [22,23]. Direct solvers such as the Gaussian elimination need $O(N^3)$ operations and $O(N^2)$ storage entries [24]. Also, it is possible that an ill-conditioned system will result in an inaccurate or divergent solution.

Although the MFS has been successfully applied to many engineering problems, it is not so easy to deal with wave problems directly, because the fundamental solutions to the wave equation are always accompanied with the Heaviside step function. When the fundamental solution including the Heaviside step function is used for interpolation, we have to face many difficult problems such as differentiating the Heaviside step function to form a linear system. In this paper, the Houbolt method is used to eliminate the time-dependent loading of system equations. And then, the given wave equation is transformed into an equivalent system of modified Helmholtz equations. The original solution to the given problem is split into the homogeneous solution and the particular solution at each time step. The newly developed sparse scheme of the MFS (SMFS) [9,25] by exploiting the natural exponential decay of the fundamental solutions of the modified Helmholtz operator is used for the fast simulation of homogeneous solution to the modified Helmholtz systems. Based on this property, the ill-conditioned dense matrix in the MFS is prevented. The particular solution is not unique. It can be obtained in different ways and particular solutions could be different [26]. In this paper, the localized method of approximate particular solutions (LMAPS) [27] proposed by Yao et al. is absorbed for the fast evaluation of particular solutions due to its localized feature to match with the local SMFS. The LMAPS is truly free from ill-conditioned dense matrix. Using the approximations in local regions, the values and their derivatives via the LMAPS can be approximated by a linear combination of nearby values. The LMAPS can be easily extended to large scale problems since the sparse matrix can be efficiently solved. The closed-form particular solutions using the Gaussian radial basis function has been used in this paper [28]. The LMAPS can be solely used without the SMFS for the non-homogeneous modified Helmholtz equations. The proposed SMFS–LMAPS is a kind of meshless method which includes the advantages of both the SMFS and the LMAPS which requires less computational time compared with the LMAPS scheme. The two-step approach (SMFS–LMAPS) requires to solve two smaller matrix systems while one-step method (LMAPS) requires to solve a larger system. Another issue concerning the optimal shape parameter c in the Gaussian radial basis function is overcome by the leave one out cross validation (LOOCV) proposed by Rippa [29]. The LOOCV method is an estimation of the generalized performance of a model trained on $n-1$ samples of data. For more details of LOOCV, we refer the readers to [30,31].

The rest of this paper is organized as follows. In Section 2, the wave equation and the Houbolt method are briefly introduced. The method of fundamental solutions in combination with the matrix sparseness technique is presented in Section 3. Then in Section 4, the localized method of approximate particular solutions is introduced. Followed by Section 5, three numerical examples are employed to verify the accuracy and efficiency. Section 5 concludes this study with some remarks.

2. Governing equations and the Houbolt method

We consider the following wave equation:

$$\nabla^2\phi - v\phi = \frac{\partial^2\phi}{\partial t^2} + f, \quad \text{in } \Omega,\ t>0, \tag{1}$$

where Ω is the bounded domain in R^2 and R^3 with its boundary $\partial\Omega$, which we shall assume to be piecewise smooth, $\phi = \phi\left(\overrightarrow{x}, t\right)$ is the physical variable, t represents the time, $\overrightarrow{x}$ is the spatial vector: $\overrightarrow{x} = [x_1,\ x_2]$ for a two-dimensional problem, $\overrightarrow{x} = [x_1,\ x_2,\ x_3]$ for a three-dimensional problem, f is given in advance, and v is a constant.

We discretize the time derivative of Eq. (1) by using the Houbolt method. The Houbolt method is a three-steps implicit and unconditionally stable time-integration scheme which is based on third-order interpolation of wave potential ϕ from a time level $t_{n-2} = (n-2)\Delta t$ to a time level $t_{n+1} = (n+1)\Delta t$. By considering a cubic curve that passes through the four successive ordinates, the following difference equation for the final derivative may be obtained:

$$\left\{\frac{\partial^2\phi}{\partial t^2}\right\}^{n+1} \approx \frac{1}{\Delta t^2}\left(2\phi^{n+1} - 5\phi^n + 4\phi^{n-1} - \phi^{n-2}\right), \tag{2}$$

where Δt is the time step size with time meshing $t_n = n\times\Delta t$, and $\phi^{n+1} = \phi\left(\overrightarrow{x}, t_{n+1}\right)$. By substituting the expression for ϕ^{n+1} from Eq. (2) into Eq. (1), we get:

$$\nabla^2\phi^{n+1} - v\phi^{n+1} - \frac{2}{\Delta t^2}\phi^{n+1} = \frac{1}{\Delta t^2}\left(-5\phi^n + 4\phi^{n-1} - \phi^{n-2}\right) + f^{n+1}. \tag{3}$$

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