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An algorithm for multi-resolution analysis of NURBS surfaces

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ABSTRACT

A new algorithm is proposed for the multi-resolution representation of NURBS (Non-Uniform Rational B-Spline) surfaces with boundary consistency constraints. The kernel idea is to employ T-meshes (T-spline control meshes) to construct the multi-resolution representation for surfaces. To this end, the original B-mesh (B-spline control mesh) is first divided into a boundary part and an inner one; then, the multi-resolution representation of the inner part is achieved by wavelet transform; at last, the topology of the boundary part is added onto the obtained multi-resolution meshes to construct the final T-spline based multi-resolution representation result. The boundary shapes keep unchanged in the whole process of the treatment, because the boundary consistency constraints are always contained in the boundary part. This advantage of the new algorithm is demonstrated by three examples.

1. Introduction

Surface modeling is widely used in different areas of computer aided engineering (CAE) such as computer aided design (CAD), animation processing, finite element analysis and computer aided manufacturing (CAM). With the fast development of CAE, researchers and engineers often have to tackle geometric models with complex shapes. However, as the complexity of the models increases, the efficiency of visualization, storage and transmission may remarkably decrease. An effective way to solve this problem is to represent complex models by wavelet-based multi-resolution technique.

Up till now, wavelet-based multi-resolution technique has been widely applied in the fields of image processing, data compression, and complex shape representation [1–4]. Multi-resolution technique of B-splines can be dated back to 1980s. In 1988, Forsey et al. first introduced hierarchical B-splines and thereby achieved multi-resolution representation for B-spline surfaces [5]. The deficiency of Forsey's method is that for a given B spline surface, it cannot give a unique multi-resolution representation result. In 2003, Narayanaswami et al. proposed a method of multi-resolution representation for curves and surfaces based on B-spline wavelets [6]. Compared with Forsey's method, the advantage of this approach is that it can give a unique multi-resolution representation result for a given surface. In 2005, Zhao et al. presented a fast variational approach for the interactive design of multi-resolution surfaces based on B-spline wavelets [4]. In 2006, Yin et al. developed a method of multi-resolution modeling for CAD surface models based on wavelets [7]. In recent years, wavelet-based multi-resolution methods have also been applied in the areas of data compression and image processing [8,9].

A common shortcoming of all the existing multi-resolution representation methods is that for one surface patch, they cannot preserve the original boundary shape at any resolution level (see Fig. 1). The current methods can impose geometric constraints in wavelet-based multi-resolution, but the result composed of multiple surface patches [4,6,7], which cannot

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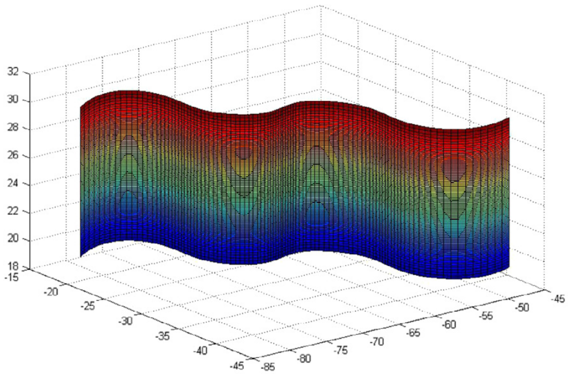
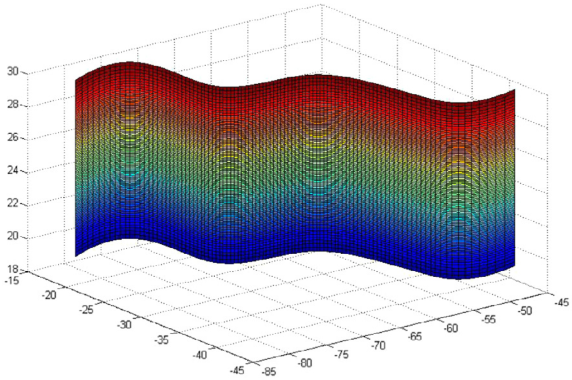
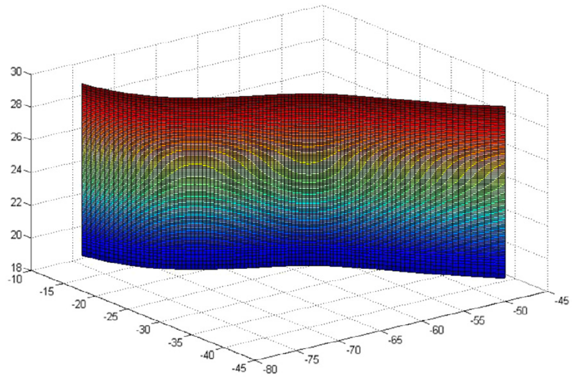
0898-1221/



(a) The original surface.



(b) The representation result at resolution level 1.



(c) The representation result at resolution level 0.

Fig. 1. Multi-resolution representation results for one surface by B-spline wavelet methods.

preserve boundary consistency constraints for one surface patch. For surface multi-resolution, the boundary consistency constraint is that the boundary should not be changed at each resolution level. This is because that all these methods represent the surface at each resolution level simply by B-spline, which is unable to characterize the boundary consistency constraints for one surface. As is known, boundary features play important roles in surface modeling. Therefore, it is necessary to develop a new multi-resolution method that can describe boundary consistency constraints and thereby preserve the original boundary shape. For this reason, the purpose of the present paper is to explore how to preserve the original boundary shape in the multi-resolution representation. To solve this problem, the T-spline technique is employed to establish a new algorithm for multi-resolution representation under boundary consistency constraints.

T-splines was proposed by Sederberg et al. in 2003 [10,11], which have T-junctions in their control meshes and so can achieve flexible local refinement. Subsequently, a number of researches were done to complete the T-spline theory [12–14] and to extend the applications of T-splines in CAD and CAE such as the isogeometric analysis based integration of NURBS or T-splines with finite element analysis [15,16].

The rest of this paper is organized as follows. Section 2 introduces the traditional wavelet-based multi-resolution representation for surfaces. Section 3 then briefly describes the T-spline surfaces. Section 4 presents the new algorithm for multi-resolution representation under boundary consistency constraints based on T-splines. Section 5 provides three examples to demonstrate the effectiveness of the new algorithm. Finally, Section 6 gives some conclusions.

2. Traditional multi-resolution representations for NURBS surfaces

An original NURBS surface $S^L(u, v)$ is defined as

$$S^L(u, v) = \frac{\sum_{i=0}^{m}\sum_{j=0}^{n} C_{i,j} w_{i,j} B_{i,d}^L(u) B_{j,d}^L(v)}{\sum_{i=0}^{m}\sum_{j=0}^{n} w_{i,j} B_{i,d}^L(u) B_{j,d}^L(v)} \tag{1}$$