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Singular boundary method for acoustic eigenanalysis

Weiwei Li [a,b], Wen Chen [a,b,*], Guofei Pang [a]

[a] State Key Laboratory of Hydrology-Water Resources and Hydraulic Engineering, International Center for Simulation Software in Engineering and Sciences, College of Mechanics and Materials, Hohai University, Nanjing 210098, China

[b] State Key Laboratory of Acoustics, Institute of Acoustics, Chinese Academy of Sciences, Beijing 100190, China

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ABSTRACT

This paper applies the singular boundary method (SBM) to two- (2D) and three-dimensional (3D) acoustics eigenproblems in simply- and multiply-connected domains. The SBM is a strong-form boundary discretization numerical method and is meshless, integration-free, and easy-to-implement. By introducing the concept of the source intensity factors, the singularity of fundamental solutions can be isolated to avoid the singular numerical integrals in the boundary element method (BEM). Similar to the BEM, the spurious eigenvalues may arise in the SBM computation. In order to extract out spurious eigenvalues from SBM results, the singular value decomposition updating techniques and the Burton–Miller method are implemented. Several 2D and 3D benchmark examples subjected to Dirichlet and Neumann boundaries are tested to examine the accuracy and stability of the present SBM strategy.

1. Introduction

The boundary element method (BEM) and boundary integral equations (BIEs) have been applied to the acoustic eigenproblems for a long time. Several schemes, such as the real-part BEM [1], imaginary-part BEM [2], and complex-value BEM [3] have been proposed for this purpose, while the spurious eigenvalues may appear in such analysis [4–6]. To deal with this drawback, the dual multiple reciprocity method [7] and the real-part dual BEM [8] are suggested to extract out the spurious eigenvalues. Some special techniques are also developed in conjunction with the BEM techniques to remedy this problem as well, such as the domain partition technique [9], the generalized singular value decomposition (GSVD) [10], singular value decomposition (SVD) techniques [11,12], and combined Helmholtz exterior integral equation formulation (CHEEF) method [13]. However, the major problem in the BEM is computationally expensive yet mathematically complex numerical integration and, in case of complex-shaped or moving boundary three-dimensional domains, time-consuming mesh generation.

In recent decades, the radial basis function (RBF) meshless methods are also applied to the acoustic eigenproblems to avoid mesh and numerical integration in the BEM schemes. Karageorghis et al. [14] used the method of fundamental solution (MFS) to the eigenanalysis of the Helmholtz equation in regular domains. Chen et al. [15–17] extended this method to the multiply-connected domain problems and investigated the problem of spurious eigensolutions. Kang et al. [18,19] proposed the non-dimensional dynamic influence function (NDIF) method to vibration analysis of membranes. Chen et al. [20,21] developed the boundary collocation method for acoustic eigenanalysis of 2D and 3D cavities. Chen et al. [22,23] applied

* Corresponding author at: State Key Laboratory of Hydrology-Water Resources and Hydraulic Engineering, International Center for Simulation Software in Engineering and Sciences, College of Mechanics and Materials, Hohai University, Nanjing 210098, China. Tel.: +86 25 83786873; fax: +86 25 83736860.
E-mail address: chenwen@hhu.edu.cn (W. Chen).

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boundary knot method to vibration analysis of plates. Chen et al. [24] used the RMM to solve multiply-connected domain eigenproblems.

The MFS [25] is a popular boundary-type RBF collocation scheme and places the source points on the fictitious boundary outside the physical domain to avoid the singularities of the fundamental solutions. However, it is not an easy task to determine appropriate location of the fictitious boundary especially for complicated domains. Several modified schemes of the MFS are proposed to eliminate the fictitious boundary, such as boundary knot method (BKM) [26], regularized meshless method (RMM) [27], singular boundary method (SBM) [28,29], boundary particle method (BPM) [30,31], and boundary distributed source method (BDSM) [32].

This study is concerned with the application of the SBM to acoustic eigenproblems directly. The SBM introduces the concept of source intensity factors to eliminate the singularities of fundamental solutions in the strong-form collocation formulation upon the coincidence of the source and collocation points on the physical boundary [33]. Unlike the BEM, the SBM avoids mesh-generation and numerical integration which being mathematically simple, computationally efficient and easy to implement. In contrast to the MFS, the SBM distributes source points on the real physical boundary and circumvents the troublesome placement of the fictitious boundary, so the method is more suitable for the complex-shaped domain problems. According to numerical experiments on potential [29], exterior acoustic [34,35], plane elasticity [36], Stokes flow [37], and heat conduction problems [38], the SBM can achieve the same order of convergence rate and accuracy as the best BEM schemes. In this study, we focus on the direct application of SBM in acoustic eigenanalysis, although the application has been investigated indirectly [34,35]. Like the BEM, the study finds that the spurious eigenvalues also occur in the SBM acoustic analysis. We employ the SVD updating techniques and the Burton–Miller method to remedy this problem.

The organization of the rest of this paper is as follows. The formulations of acoustic eigenproblems using the SBM are presented in Section 2, followed by Section 3 remedying the spurious eigensolutions problem. In Section 4, the accuracy and validity of the SBM are examined through 2D and 3D benchmark cases. Finally, some conclusions are summarized in Section 5.

2. The SBM formulation of acoustic eigenproblems

2.1. Description of the acoustic eigenproblems

The governing equation of an acoustic eigenproblem is the Helmholtz equation as follows:

$$\left(\nabla^2 + k^2\right) u(x) = 0, \quad x \in \Omega, \tag{1}$$

where u is the acoustic pressure field, k the wave number, and Ω the domain of the problem. The boundary conditions can be Dirichlet, Neumann or Robin type.

2.2. Singular boundary method

The SBM interpolation of the above Helmholtz equation (1) is expressed as

$$u(x_i) = \sum_{j=1, j\neq i}^{N} \alpha_j G(x_i, s_j) + \alpha_i U_S^{ii}, \quad i = 1, \ldots, N, \tag{2}$$

$$q(x_i) = \frac{\partial u(x_i)}{\partial n_x} = \sum_{j=1, j\neq i}^{N} \alpha_j \frac{\partial G(x_i, s_j)}{\partial n_x} + \alpha_i Q_S^{ii}, \quad i = 1, \ldots, N, \tag{3}$$

where x_i and s_j denote the ith collocation and the jth source points, respectively, as shown in Fig. 1, α_j is the jth unknown coefficient, N the number of source points, and n_x the outward unit normal on the collocation points x_i. U_S^{ii} and Q_S^{ii} are the source intensity factors to avoid the fundamental solution singularities when collocation and source points coincide. The fundamental solutions of 2D and 3D problems are written by

$$G(x_i, s_j) = iH_0^{(1)}(kr_{ij})/4 \tag{4}$$

and

$$G(x_i, s_j) = \exp(ikr_{ij})/(4\pi r_{ij}), \tag{5}$$

respectively, where $H_n^{(1)}$ is the nth order Hankel function of the first kind, and $r_{ij} = \left|x_i - s_j\right|$ is the Euclidean distance between the source and collocation points.

The key issue in the SBM is the evaluation of the source intensity factors. For the sake of limited space, this paper only gives the formulations for determining these factors and refers the readers to literature [28,29,34,36,38] for more details.