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Nonlinear thermo-elastic bending behavior of graphene sheets embedded in an elastic medium based on nonlocal elasticity theory

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ABSTRACT

This paper investigates the large deflection behavior of orthotropic single layered graphene sheet (SLGS) embedded in a Winkler–Pasternak elastic medium under a uniform transverse load in thermal environments. Using the nonlocal differential constitutive relations of Eringen, the SLGS is modeled as a nonlocal orthotropic plate. Using the principle of virtual work, the coupled nonlinear equilibrium equations are obtained based on first-order shear deformation theory (FSDT) and the von Kármán geometrical model. The differential quadrature (DQ) discretized form of the governing equations with clamped and simply supported boundary conditions is derived. The Newton–Raphson iterative scheme is used to solve the resulting system of nonlinear algebraic equations. Effects of small scale parameter, thermal environment, width-to-length elasticity ratio, aspect ratio, thickness, elastic foundation, load value and boundary conditions are considered in detail. The results show that, unlike the simply supported boundary conditions, increase of small scale parameter plays a decreasing role in effect of thermal environment on the deflections of nanoplates with clamped edges. It is also observed that increasing the plate thickness, thermal load effects decline noticeably and depending on the small scale value the thermal loads do not have any significant effect on the results beyond a specified thickness.

1. Introduction

In recent years, nanoplates such as Graphene sheets (GSs) have attracted a great deal of attention from the researchers community for their superior properties and extensive applications in many fields such as modern aerospace, superfast microelectronics, micro- or nano-electromechanical systems (MEMS or NEMS), biomedical, bioelectrical, and nano-composites [1–5]. Single layered graphene sheets (SLGSs) are defined as a flat one-atom-thick carbon tightly packed into a two-dimensional honeycomb lattice. Since the most potential applications of the SLGSs depend on our understanding of their mechanical behavior, the mechanical analysis of nanoplates has become a subject of primary interest in recent studies [6]. The traditional continuum theory is a scale free theory and thus cannot predict the mechanical behaviors of nanostructures properly [7]. Recently, size-dependent continuum modeling of nanostructures has received great deal of attention of scientific community because controlled experiments on nanoscale are difficult to perform and molecular dynamic simulations are highly computationally expensive and are not suitable for analyzing large scale systems. There are various size-dependent continuum theories such as couple stress theory [8], strain gradient elasticity theory [9,10], modified

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couple stress theory [11,12] and nonlocal elasticity theory [13–15]. Among these theories, nonlocal elasticity theory has been widely applied [16–23]. Based on this theory, a lot of works have been so far devoted to the study of buckling and vibration behaviors of SLGSs as well as their properties [24–39]. However, less theoretical studies have been carried out on the bending behavior of these 2D atomic structures. Reddy [40] reformulated the bending of classical and shear deformation beam and plate theories using the nonlocal differential constitutive relations of Eringen and the von Kármán nonlinear strains. Using the Navier solution, Wang and Li [41] studied small deflection behavior of the isotropic nanoplate embedded in an elastic matrix based on the nonlocal Mindlin and Kirchhoff plate models. Also, Aghababaei and Reddy [42] presented the Navier solutions for bending and vibration of simply supported nano-plates based on third-order shear deformation plate theory (TSDT) and nonlocal linear elasticity theory of Eringen. However, Golmakani and Rezatalab [43] proved that the Navier solution cannot predict acceptable bending responses of nanoplates subjected to uniform transverse loading based on the nonlocal theory of Eringen. This is because of an inaccurate approximation of the load derivation in terms of displacement based on the Navier solution formed as a result of the small scale parameter in classical relations. Recently, using an approximate solution, Kemal Baltacıoglu et al. [44] studied the nonlinear static response of laminated composite plates by discrete singular convolution method. They presented the governing equation for bending based on first-order shear deformation theory in the von Kármán sense. Shen et al. [45] studied the nonlinear bending response of SLGSs subjected to a transverse uniform load in thermal environments on the basis of a nonlocal classical plate theory (CPT) with a von Kármán type of kinematic nonlinearity. Xu et al. [46] investigated the nonlinear bending behavior of a bi-layered rectangular graphene sheet subjected to a transverse uniform load in thermal environments, using the nonlocal CPT including the van der Waals interactions. However, in these studies all four edges are assumed to be simply supported and also they did not study the effects of elastic medium. Moreover, using a two-step perturbation technique Shen [47] investigated postbuckling, nonlinear bending and nonlinear vibration behaviors of a simply supported orthotropic nanoplate resting on a two-parameter elastic foundation in thermal environments based on nonlocal CPT. More recently, using the differential quadrature method (DQM) and (GDQM) for different boundary conditions, Golmakani and Rezatalab [48] investigated the nonlinear bending behavior of the orthotropic SLGSs in an elastic matrix subjected to a transverse uniform load with various boundary conditions based on nonlocal first-order shear deformation theory (FSDT) and nonlocal CPT using DQM. Heydari et al. [49] studied the Nonlinear bending behavior of orthotropic Mindlin plate resting on orthotropic Pasternak foundation using GDQM. They derived the governing equations based on Hamilton's principle and solved them when all four ends are clamped. They found that considering elastic medium decreases deflection of the plate, and the effect of the Pasternak-type is higher than the Winkler-type on the maximum deflection of the plate. Dastjerdi and Jabbarzadeh [50] deal with the use of the nonlocal first-order plate theory for nonlinear bending analysis of bilayer orthotropic graphene sheets resting on Winkler–Pasternak elastic foundation.

According to the best knowledge of authors, there is still no literature dealing with the nonlinear thermo-mechanical bending behavior of orthotropic nanoplate with different boundary conditions based on nonlocal FSDT. Using the principle of virtual work, the nanoplate equilibrium equations are derived in terms of the generalized displacements based on FSDT using the nonlocal differential constitutive relations of Eringen and the von Kármán nonlinear strains. Differential quadrature method is used to solve the governing equations for simply supported and clamped boundary conditions and various combinations of them. To verify the present results and formulations, some comparison studies are carried out between the obtained results and the available solutions in the literature. Excellent agreement between the obtained and available results is observed. Finally, the small scale effects on the bending behavior of nanoplates are investigated through considering various parameters such as small scale parameter, boundary conditions, width-to-length elasticity ratio, Winkler and Pasternak elastic foundations, aspect ratio, thickness of plate, load value and thermal environmental conditions.

2. Governing equations

The SLGS is modeled as a rectangular nanoplate and the elastic medium is defined by a two-parameter Pasternak elastic foundation. Fig. 1 shows the idealized model and continuum model used in this study for a SLGS resting on two-parameter foundation with length l_x, width l_y and thickness h parallel to the x, y and z axes of the Cartesian coordinates frame, respectively. Also, as shown in Fig. 1 the nanoplate is subjected to thermal environmental conditions and uniform transverse load q. According to the nonlocal continuum theory of Eringen [13–15] which accounts for the small scale effects by assuming the stress at a reference point as a function of the strain field at all neighbor points in the continuum body. The components of nonlocal stress tensor $\sigma_{ij}(x)$ are given in the form of the following expression [14]:

$$\sigma_{ij}(X) = \int \lambda \left(\left| X - X' \right|, \alpha \right) \tau_{ij}\left(X' \right) dV \left(x' \right) \quad \forall x \in V. \tag{1}$$

Here, σ_{ij}, τ_{ij} are the nonlocal and classical or local stress tensor, respectively and $\lambda\left(\left|X - X'\right|, \alpha\right)$ is the nonlocal modulus function which describes the strain effect at X point for stress at reference point X'. It can be observed that the integral constitutive relation (1) makes the elasticity problems difficult to solve. Therefore, the following differential form of the nonlocal constitutive equation is defined by Eringen [14]:

$$\left(1 - \mu \nabla^2\right) \sigma^{NL} = \sigma^{L} \tag{2}$$