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Application of Differential Quadrature method in free vibration analysis of nanobeams based on various nonlocal theories

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ABSTRACT

Conducting experiments with nanoscale size is quite difficult to handle. In this regard, mathematical modeling and their solutions play a vital role in the field of nanotechnology. In the present work, the governing differential equations are converted into a single unknown function and then Differential Quadrature (DQ) method is employed to investigate free vibration of nanobeams based on different beam theories like Euler–Bernoulli, Timoshenko, Reddy and Levinson in conjunction with nonlocal elasticity theory for the first time. Application of DQ method in the governing differential equation converts the problem to a generalized eigenvalue problem and the solution of this gives the frequency parameters. It is observed that frequency parameters are overpredicted in Euler–Bernoulli beam theory than other types of beam theories. It is also concluded that the effect of nonlocal parameter is more in case of higher vibration modes. Present results are compared with other available results and are found to be in good agreement.

1. Introduction

Recently nanomaterials have aroused interest among the scientific communities in the field of physics, chemistry and engineering. These materials have special properties [1] like mechanical, electrical and electronic. Because of these properties, nanomaterials are found to be the components of various nanoelectromechanical systems and nanocomposites. Some of these materials are nanoparticles, nanowires, nanotubes, nanotube resonators [2], nanoactuators [3] etc. Among these, beam type structures play a vital role in the field of nanotechnology. Beam type structures are widely used in civil, mechanical and aerospace engineering. Recently they have been used in nanoelectromechanical and microelectromechanical systems.

Because of its importance in practical life, dynamic analysis of beam structures has been carried out by the researchers. One should have proper understanding about the mechanical behaviors of nanobeams for accurate prediction of vibration characteristics. So far, three approaches have been developed to study mechanical behaviors. These are atomistic, semi-continuum and continuum models. Continuum mechanics are classified into classical continuum mechanics and nonclassical continuum mechanics. In classical continuum model, lattice spacing between individual atoms is not taken into consideration. As a result, the significant influence caused by small scale effects such as electric force, chemical bond and van der waals force is neglected when classical continuum model is considered. Both experimental and atomistic simulation results show that at nanoscale, the small length scale effect (such as lattice spacing between individual atoms) may not be neglected. Hence various nonclassical continuum theories like strain gradient theory, couple stress theory, micropolar theory

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and nonlocal elasticity theory have been developed to incorporate size effect by introducing an intrinsic length scale. Among these theories, nonlocal elasticity theory proposed by Eringen [4] has been widely applied in the vibration of nanobeams. The small scale effect depends on the crystal structure in lattice dynamics and the nature of physics under investigation. Conducting experiments with nanoscale size is quite difficult and expensive. Therefore, development of appropriate mathematical models for vibration analysis of nanobeams is an important issue concerning its applications. In this respect, authors have investigated various studies either by numerically or by analytically. The value of small scale effect which plays a vital role in the nonlocal elasticity theory has been calibrated using molecular dynamics [5,6]. When the size of beam is of nanoscale dimension, nonlocal impact plays a significant role in the prediction of natural frequencies and vibrating modes especially higher order natural frequencies and vibrating modes [7]. One may find significant role of nonlocal effects in nanoscale devices [8,9]. Analytical results of Euler–Bernoulli and Timoshenko nanobeams show that frequency parameters decrease with increase in scaling effect parameter [10]. Reddy [11] has shown analytical solutions for various beam theories like Euler–Bernoulli, Timoshenko, Reddy and Levinson in case of simply supported boundary condition. Aydogdu [12] has given a general expression for the displacement fields of all the well known beam theories and analytical solution for simply supported boundary condition has also been presented. Some numerical methods like meshless [13], Differential Quadrature [14,15] and Rayleigh–Ritz have also been applied in various studies related with vibration of nanobeams. It is not always possible to find analytical solutions for all set of boundary conditions at the edges. In this regard, Rayleigh–Ritz method is an efficient numerical method in handling all set of classical boundary conditions. Chebyshev polynomials and boundary characteristic orthogonal polynomials have been used in the Rayleigh–Ritz method to study vibration of Timoshenko nanobeams [16,17]. Authors have also considered various complicating effects in the vibration of nanobeams based on nonlocal elasticity theory. Loya et al. [18] have shown effects of nonlocal parameter on the frequency parameters when a nanobeam is divided into two segments and are connected by rotational spring at the cracked position. Zhang et al. [19] investigated double walled carbon nanotubes to examine the influence of nonlocal parameter on the natural frequencies. Double nanobeam systems are useful in nano-optomechanical systems and sensor applications. Seeing practical applications, authors have studied vibration of double nanobeam systems and the study shows that small scale effects are higher with increasing nonlocal parameter in the in-phase vibration than in the out-of-phase vibration. This is due to the fact that the stiffness of the springs in out-of-phase vibration reduces the nonlocal effects [20]. Ansari et al. [21] studied nonlinear vibration characteristics of multiwalled carbon nanotubes embedded in surrounding elastic medium. A detailed study has also been conducted to analyze the influences of nonlocal parameter, length of the tubes, spring constant and end supports on the nonlinear free vibration characteristics of Single Walled Carbon NanoTubes [22] and Double Walled Carbon NanoTubes [23]. Eltaher et al. [24] studied free vibration of functionally graded size dependent nanobeams.

Since conducting experiments with nanoscale size is quite difficult to handle, so efforts are always being done by the researchers to develop efficient numerical or analytical methods for obtaining better results. The literature reveals that most of the studies have been carried out by using analytical methods for simply supported boundary condition. However, it is not always possible to obtain analytical solutions for complicated geometries. Some of the numerical methods such as finite element method and Rayleigh–Ritz method have been employed by the researchers. For applying these methods, one must have sound knowledge about variational principles. Again, subsequent application of variational principles often requires a proper understanding of principles of mechanics. This has motivated the search for an approximate computational technique. In this context, one may use differential quadrature method.

Differential Quadrature (DQ) method has been introduced by Bellman and Casti [25] for the first time. This is an efficient numerical method for the solution of linear and nonlinear partial differential equations. Then Bert et al. [26] applied DQ method in structural problems. Since then, researchers are investigating linear and nonlinear structural problems using DQ method. Different procedure has been used by the authors to implement boundary conditions in the DQ method. Firstly, δ technique was proposed by Bert et al. [27] to implement boundary conditions. In this procedure [28], one boundary condition is implemented at the boundary point and other boundary condition at a distance δ from the boundary point. There are two major drawbacks of this approach. Firstly, since implementation of boundary condition at the δ point is an approximation of the true boundary condition which should be implemented at the boundary, therefore accurate numerical solutions lies on smaller value of δ. Secondly, smaller value of δ causes the solutions to oscillate since weighting coefficient matrices become highly ill conditioned. The above mentioned approach is suitable for clamped end but not suitable for simply supported and simply supported–clamped ends. Seeing demerits of this approach, Bert [29–34] proposed a new approach in applying boundary conditions. In this approach, only one boundary condition is numerically implemented and the other boundary condition is built into the DQ weighting coefficient matrices. Following are some of the advantages of this approach.

(i) Boundary conditions are properly satisfied since they are applied at the boundary points. Therefore, better results may be obtained. (ii) The effect of δ on the results is eliminated. (iii) Excellent results are obtained with less computational effort.

On the other hand, though different beam theories have been employed in vibration of nanobeams but analytical or numerical solutions are not yet found for some of the boundary conditions. Also detailed investigation has not yet been carried out for Reddy beam theory and Levinson beam theory.

Though, some of the studies have been done using DQ method, but to the best of author's knowledge, the article provides first time for computing frequency parameters of free vibration of nanobeams based on four different beam theories with the application of differential quadrature method by substituting boundary conditions in the coefficient matrix. In this article, governing differential equations have been converted into a single variable and then DQ approach is utilized to solve those equations. Beam theories in conjunction with nonlocal elasticity theory have been applied to illustrate the effect of boundary