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An efficient meshless technique for the solution of transversely isotropic two-dimensional piezoelectricity

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ABSTRACT

In this paper, a new meshless technique is presented for the solution of two-dimensional piezoelectricity problem. The technique derivation is based on the solution of the corresponding analog equation by transforming the original set of differential equations into three Poisson equations with unknown right hand side terms. Boundary discretisation of the resulting integral equations is eliminated by the use of the method of fundamental solutions. The right hand side terms are represented in the solution as particular solutions expressed in terms of radial basis functions. The problem solution is then rewritten in its new form, which involves complementary solution and particular solution. The governing partial differential operator for piezoelectricity is applied on the obtained solution form and forced to be satisfied at a set of domain points, whereas the prescribed boundary conditions are satisfied at another set of boundary points. The proposed technique is implemented into computer code where several numerical examples with different boundary conditions are tested. The results demonstrated excellent agreement with those obtained from analytical and FEM solutions.

1. Introduction

Piezoelectric materials recently demonstrated high potential among application for their ability of coupling electrical and mechanical forms of energy. Actuators [1], sensors [2], transducers [3] are common examples of such applications. Their use had been extended also to energy harvesting projects for power generation [4–6]. Their accurate coupling, small size and low cost gave them a unique advantage over other devices in micro [4,7] and nano [6] applications.

The constitutive equations representing the piezoelectricity have been modelled using several techniques. An early work was done by Deeg [8] who derived electro-elastic fundamental solutions for 3D piezoelasticity using the Radon transform. It was then used in the BEM by several researchers such as Sanz et al. [9] for fracture mechanics in three dimensional analysis. Zu [10] used Radon transform in combination with Fourier transform in order to derive fundamental solution for 2D piezoelasticity. Lee and Jiang [11] used the double Fourier transform to derive a fundamental solution expression for 2D transversely isotropic piezoelasticity for the BEM. Ding et al. [12] derived fundamental solutions using harmonic functions and simplified the solutions in the case of distinct eigenvalues which was then extended by Ding and Jiang [13] who derived the fundamental solution for three cases of piezoelectric infinite media and gave three groups of expressions of

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the harmonic functions. It can be generally noted that the derived expressions contain several terms to be evaluated which adds high computational effort when implemented and besides, some of them lacks generality. Jin et al. [14] transformed the piezoelasticity governing equations into three generalised Laplace equations and used generalised complex variables to obtain the solution expressions. They [14] used the developed solutions as trial functions in the Trefftz collocation and Trefftz Galerkin formulations. A few researchers used meshless techniques for the analysis of piezoelectric materials such as Chen et al. [15] who modelled antiplane piezoelectricity problems with multiple inclusions by employing a regularised meshless method.

The technique proposed in this paper for modelling piezoelectricity is based on the Analog Equation Method (AEM) and the Method of Fundamental Solutions (MFS). The AEM was used by several researchers for various problems such as Wang and Qin [16] who used it in combination with the virtual boundary collocation method to analyse generalised linear and nonlinear Poisson-type problems. Ishiguro et al. [17] used the AEM to analyse steady state heat conduction in anisotropic solids. Sapountzakis and Kampitsis [18] used it for nonlinear analysis of shear deformable beam–columns. Hu et al. [19] solved a heat transfer problem of molten polymer flow. Kokkinos [20] used the AEM for layer-wise thick plates modelling.

The MFS, on the other hand, was introduced by Kupradze and Aleksidze [21]. It was then implemented in the solution of several boundary type problems. Karageorghis and Fairweather used the MFS for the solution of the biharmonic equation [22], and for axisymmetric elasticity problems [23]. Alves et al. used the MFS for Poisson problems [24]. Fam and Rashed derived a dipole formulation for the MFS for potential problems [25] and used the particular solution technique for the solution of elastostatic 3D problems with body forces within the MFS context [26]. The convergence of the method was studied by Mitic and Rashed [27]. Advanced formulations for the MFS were proposed in order to overcome some limitations of this technique such as the adaptive refinement algorithm proposed by Saavedra and Power for the solution of Laplace Problems [28], and the continuous collocation introduced by Fam and Rashed for the solution of three-dimensional elastostatics [29]. It was generally noticed that the performance of the MFS in the solution of Laplace equation in potential problems is better in comparison to its performance with Navier equation in elastostatics [30], therefore, it was expected to be suitable for the AEM formulation presented in this paper.

In this paper, a new meshless technique, based on the MFS formulation and simplified via the AEM technique, is derived for the solution of planar piezoelectric problems. The AEM is used in the presented formulation for the purpose of transforming the coupled electro-mechanical governing equations into three uncoupled fictitious analog equations. Each analog equation can be modelled as a homogeneous part with known fundamental solution of same order as the original governing equation and a particular solution representing the unknown terms of the fictitious analog equation. The homogeneous solution is then represented by means of the MFS in order to avoid both boundary discretisation and the involved singular integrals. The unknown coefficients are determined through the satisfaction of the governing equations at the domain points and the prescribed conditions along the domain boundaries. Finally, it is demonstrated via several examples that the proposed technique can be simply implemented and produces highly accurate results.

2. Governing equations for piezoelectricity

The constitutive equations for two-dimensional transversely isotropic piezoelectric materials [10] (taking x_2 as the polarisation direction) can be rewritten as follows:

$$\begin{Bmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{12}\\ D_1\\ D_2\end{Bmatrix}=\begin{bmatrix}c_{11} & c_{12} & 0 & 0 & -e_{21}\\ c_{12} & c_{22} & 0 & 0 & -e_{22}\\ 0 & 0 & c_{33} & -e_{13} & 0\\ 0 & 0 & e_{13} & \varepsilon_{11} & 0\\ e_{21} & e_{22} & 0 & 0 & \varepsilon_{22}\end{bmatrix}\begin{Bmatrix}s_{11}\\ s_{22}\\ 2s_{12}\\ E_1\\ E_2\end{Bmatrix} \tag{1}$$

where E_ℓ is the electric field, $s_{k\ell}$ is the mechanical strain, c_{ij} is the stiffness matrix coefficients, $e_{j\ell}$ is the piezoelectric tensor coefficients and $\varepsilon_{k\ell}$ is the dielectric tensor coefficients. The relationship between the mechanical strain and the mechanical displacement can be expressed as:

$$s_{k\ell}=\frac{1}{2}(u_{k,\ell}+u_{\ell,k}) \tag{2}$$

whereas for electric potential and electric field:

$$E_\ell=-\varphi_{,\ell}. \tag{3}$$

The governing equations for piezoelectricity are expressed as follows [31]:

$$\sigma_{ij,j}=-f_i \tag{4}$$

$$D_{j,j}=-q \tag{5}$$

where σ_{ij} is the mechanical stress, f_i is the mechanical body force term, D_j is the electric displacement and q is electric charge density per unit volume.