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An a posteriori-based, fully adaptive algorithm with adaptive stopping criteria and mesh refinement for thermal multiphase compositional flows in porous media ☆

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ABSTRACT

In this work we develop an a posteriori-based adaptive algorithm for thermal multiphase compositional flows in porous media. The key ingredients are fully computable a posteriori error estimates, bounding the dual norm of the residual supplemented by a nonconformity evaluation term. The theory hinges on assumptions that allow the application to variety of discretization methods. The estimators are then elaborated to estimate separately the space, time, linearization, and algebraic errors. This additional information is used to formulate a fully adaptive algorithm including adaptive stopping criteria for iterative solvers as well as refinement/derefinement criteria for both the time step and the mesh size. Numerical validation is provided on an industrial case study in the context of oil-recovery based on the steam-assisted gravity drainage procedure. Implicit cell-centered finite volumes with phase-upwind and two-point discretization of the diffusive fluxes are considered. It is shown that significant gains in computational cost can be achieved in this example, without hindering the quality of the results as measured by quantities of engineering interest.

1. Introduction

The thermal multiphase compositional model in porous medium describes the flow of several fluids through a subsurface under a non-isothermal condition. The governing equations are the conservation of the amount of each component and the conservation of energy, which are partial differential equations, supplemented by algebraic equations expressing the conservation of volume, the conservation of the quantity of matter, and the thermodynamic equilibrium, see [1–3].

Thermal models are especially important for simulation of the enhanced oil recovery, where the increase of temperature reduces the oil viscosity which in turn improves mobility and makes the production easier, therefore leading to better

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recovery indices. Several recent works deal with the simulation of thermal oil-recovery, like, e.g., [4–11]. Thermal processes play also an important role in the modeling of geothermal reservoirs, see, e.g., [12] and the references therein.

A mathematical structure of multiphase thermal models of flow in porous media is proposed in [13]. The authors derive and numerically solve a system of PDEs modeling multicomponent, two-phase, thermal fluid flow in porous media. For this purpose, they develop an algorithm that aims at a balance between stability and accuracy. This approach was used previously for reservoir simulation of black-oil model [14] and also for compositional models [15]. Recently, it has been proposed in [11,16] to formulate the phase transitions as a set of local inequality constraints and use the complementarity approach.

Many numerical methods have been proposed for the discretization of the multiphase compositional model: finite differences and finite element methods in, e.g., [1,17–19], mixed finite element methods in, e.g., [20–23], finite volume methods in, e.g., [24–29], and recently vertex-centered methods on general 3D meshes in [30]. In the simulation of oil recovery processes based on the injection of a hot fluid to reduce oil viscosity, a key point is to track the evolution of the saturation front. Since the location of the front evolves in time, adaptive mesh refinement is mandatory to make computations accessible. Algorithms for adaptive mesh refinement have been considered, cf. [5,31,32] for dynamic griding in thermal and isothermal models, and other recent contributions, cf. [6–9,33–37].

In this work, we go one step further with the reduction of the computational cost by proposing a fully-adaptive algorithm based on a posteriori error estimates. As a matter of fact, the discretization of the thermal compositional model leads to a nonlinear, strongly coupled system of algebraic equations, whose resolution demands a significant computational effort even when the mesh is adaptively refined. The key idea is to develop a posteriori error estimators allowing to distinguish the different components of the error, and use them to formulate stopping criteria for the iterative algebraic and nonlinear solvers together with refinement/derefinement criteria for both the time step and the space mesh. The a posteriori error estimators are derived following the general ideas of [38–40], where they are used to formulate adaptive stopping criteria (without mesh adaptation) for the isothermal case. The main novelties of the present work with respect to [40] are

(i) the extension of the framework to the thermal case accounting for one additional equation expressing the energy balance. From a practical viewpoint, the main difficulty is that adding a dependence on temperature to the physical parameters embeds strongly nonlinear mechanisms in the model;
(ii) the application to adaptive mesh refinement in the context of a three-dimensional industrial case study. This is a significant step to assess the performance of the overall cost-reduction strategy and makes it more appealing for practitioners.

Other work which deserves being recognized at this point is [41], where a rigorous a posteriori error analysis for the immiscible incompressible two-phase flow is developed under the assumption that the flow process is isothermal.

The present paper is organized as follows. Section 2 details the unknowns and the physical properties related to the general thermal multiphase compositional model and describes the governing equations that constitute the mathematical system of the model. In Section 3 we consider a discretization of the thermal model based on the two-point finite volume scheme in space and the backward Euler scheme in time. Linearization by the Newton method and algebraic resolution by an arbitrary iterative solver is also discussed. In Section 4 we postprocess the original phase pressures and temperature and we devise their $H_0^1(\Omega)$-conforming reconstructions, as well as $\mathbf{H}(\mathrm{div};\Omega)$-conforming fluxes needed in the a posteriori analysis. In Section 5 we introduce the weak formulation of the problem, define the corresponding error measure, and derive the a posteriori error estimate. Section 6 finally illustrates the numerical results on an enhanced oil recovery thermal process for heavy oil. We show results corresponding to adaptive mesh refinement strategy saving an important number of mesh cells during the simulation, without affecting the precision of the resolution.

2. The thermal multiphase compositional model

We consider the flow through a porous medium of several fluid phases, each composed of a finite number of components from a given set. Mass exchange between phases as well as thermal effects are accounted for. The precise formulation we use extends that of Eymard, Guichard, Herbin, and Masson [30] based on the original paper of Coats [2]; see also [42] and [40].

Let $\Omega \subset \mathbb{R}^d$, $d \geq 1$, denote a bounded connected polygonal domain with boundary $\partial\Omega$, and let $t_F > 0$. In petroleum-related applications, Ω typically represents a reservoir, while t_F is the simulation time. We denote by $\mathcal{P} = \{p\}$ and $\mathcal{C} = \{c\}$ respectively the set of phases and components. A synthetic description of the fluid system is given in terms of the component-phase matrix $\mathbf{M} = [m_{cp}]_{c\in\mathcal{C},\, p\in\mathcal{P}} \in \{0,1\}^{\mathcal{C},\mathcal{P}}$ such that, for all $c \in \mathcal{C}$ and all $p \in \mathcal{P}$, $m_{cp} = 1$ if the component c is contained in the phase p and 0 otherwise. Given the component-phase matrix, we can define, for all $p \in \mathcal{P}$, the set of components present in the phase p as $\mathcal{C}_p = \{c \in \mathcal{C};\ m_{cp} = 1\}$. Conversely, for each component $c \in \mathcal{C}$, the set of phases containing c is given by $\mathcal{P}_c = \{p \in \mathcal{P};\ m_{cp} = 1\}$.

2.1. Unknowns

The unknowns of the model are

(i) the *reference pressure* P;
(ii) the *temperature* T;