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An alternative lattice Boltzmann model for three-dimensional incompressible flow

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ABSTRACT

In this work, an alternative lattice Boltzmann (LB) model for three-dimensional (3D) incompressible flow is proposed. The equilibrium distribution function (EDF) of the present model is directly derived in accordance with the incompressibility conditions by applying the Hermite expansion. Moreover, an alternative formula for pressure computation is designed from the second order moment of the distribution function. The present 3D LB model inherits the advantageous features of Guo's LB model: the density is a constant, the fluid pressure is independent of density and the Navier–Stokes (N–S) equations for incompressible flow can be derived. Two benchmark tests, flow over a backward-facing step and the lid-driven cavity flow, are applied to validate the present model. Accurate results for these tests are obtained with the present model, and further comparisons with the previous LB models (the standard LB model, the He–Luo model and Guo's LB model) demonstrate that the present model provides better accuracy in the region of high deviatoric stress and such advantage is further enhanced by using the D3Q27 lattice.

1. Introduction

The lattice Boltzmann method (LBM) has won great popularity owing to its successful application in complex fluids such as suspensions and interfacial dynamics [1–6]. The lattice Boltzmann equation is regarded as an explicit finite difference approximation of a velocity-discrete Boltzmann equation [7,8]. In describing the macroscopic Navier–Stokes (N–S) equations, it is second order accurate in space and time [1,9,10]. In particular, the spatial second order accuracy for the deviatoric stress tensor is proved theoretically by Luo and Yong [1]. In addition, comparing with the second order finite difference method for the direct discretization of the N–S equations, the LBM has advantages in the low numerical dissipation and better isotropy [1,10,11].

Based on the Chapman–Enskog analysis [12], the standard LB models can recover the isothermal, compressible N–S equations, approximating the incompressible N–S equations with second order error in Mach number (Ma) [10]. In particular, in the standard LB model, the fluid pressure is represented by the density, and thus the resulting continuity equation departs

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from the one pertaining to the incompressible flow. Such violation of the incompressibility may produce some serious errors in simulating incompressible flow [13–16].

To avoid or reduce such discrepancy of the standard LB model, many attempts have been devoted to building an incompressible LB model. By assuming the density relating to the fluid momentum to be a constant, an incompressible LB model for steady flow is designed by Zou et al. [14]. Similarly, an incompressible LB model, in which the fluid density ρ is split up into a constant mean density ρ_0 and a small fluctuation $\delta\rho$ and then the effect of $\delta\rho\mathbf{u}$ is neglected, is proposed by He and Luo (notated as He–Luo model) [15]. Although it is claimed that the He–Luo model is applicable for both steady and unsteady flow, the density fluctuation in this model is related to the pressure and the compressibility effect remains second order in Ma for unsteady incompressible flow [10]. Differing from all the above-mentioned LB models, the fluid density in Guo's model is prescribed to be a constant and the pressure is independent of the density [13], therefore, the requirements of the incompressibility are satisfied. However, an approximation relating to the deviatoric stress tensor is introduced during the pressure computation in Guo's LB model, which may affect the accuracy in the region of high deviatoric stress.

Despite of the demerits of above models, accurate results are obtained when applying them to some certain incompressible flow [13–18], such as the fully developed flow in a duct or a pipe, lid-driven cavity flow with low Reynolds number (Re) and flow over obstacles. However, when referring to flow with severe convection, such as the lid-driven cavity flow with higher Re, the visible discrepancies of the velocity near the no-slip boundary are reported in both 2D and 3D LB simulations [19–25].

Inspired by Guo's model, an alternative 3D LB model for incompressible flow is constructed to better cope with the incompressibility conditions and the approximation errors in regions with high deviatoric stress. From the Chapman–Enskog analysis, it is reasonable to conclude that the specific form of the relevant macroscopic equations depends on the moments of the EDF. By assuming the pressure to be independent of the density and setting the fluid density to be a constant, the moments of the distribution function are modified directly. Then, Grad's Hermite tensorial polynomials [26,27] are utilized to derive the continuous expression of the EDF. Applying the discrete velocity vectors of the D3Q19 and D3Q27 lattice models [28,29], the continuous EDF is discretized. Afterwards, the pressure formula, which is alternatively different from other LB models, is designed from the modified second order moment of the distribution function. Moreover, considering the specific expression of the discrete EDF, the pressure formula is further revised to improve the convergence properties of the present model. To validate the present model, the simulations of the backward-facing step flow and the 3D lid-driven cavity flow are performed, and the simulated results are compared with the reference solutions in the literature [30–33].

2. The 3D incompressible LB model

In this section, some common LBM features, the governing equation and the discrete velocity vectors of the D3Q19 lattice model, are given first. Then the theoretical properties of the standard LB model, the He–Luo model and Guo's model are presented. Finally, the detailed derivations of the present 3D incompressible LB model are provided.

2.1. The lattice Boltzmann equation

The lattice Boltzmann equation is

$$f_\alpha(\mathbf{x}+\boldsymbol{\xi}_\alpha dt, t+dt) - f_\alpha(\mathbf{x}, t) = -\frac{1}{\tau}[f_\alpha(\mathbf{x}, t) - f_\alpha^{(0)}(\mathbf{x}, t)] \tag{1}$$

where τ is the single dimensionless relaxation parameter of the BGK collision operator [34]; $f_\alpha(\mathbf{x}, t)$ and $f_\alpha^{(0)}(\mathbf{x}, t)$ are the discrete distribution function and its equilibrium version, respectively. The BGK collision operator is adopted in all the LB simulations in this work.

The discrete velocity vectors $\boldsymbol{\xi}_\alpha$ in the D3Q19 lattice, which is widely adopted in the 3D LB simulations [24,29], are defined as below:

$$\boldsymbol{\xi}_\alpha = \begin{cases} (0,0,0), & \alpha = 0 \\ (\pm1,0,0)c,\ (0,\pm1,0)c,\ (0,0,\pm1)c, & \alpha = 1\text{–}6 \\ (\pm1,\pm1,0)c,\ (\pm1,0,\pm1)c,\ (0,\pm1,\pm1)c, & \alpha = 7\text{–}18 \end{cases} \tag{2}$$

where the lattice constant $c = dx/dt$, and dx, dt are the mesh size and time step, respectively. The relating weight coefficient w_α is

$$w_\alpha = \begin{cases} 1/3 & \alpha = 0 \\ 1/18 & \alpha = 1\text{–}6 \\ 1/36 & \alpha = 7\text{–}18. \end{cases} \tag{3}$$

2.2. The standard LB model

In the standard model [35], the expression of the discrete EDF $f_\alpha^{(0)}$ is determined as

$$f_\alpha^{(0)} = \rho w_\alpha \left\{ 1 + \frac{\boldsymbol{\xi}_\alpha \bullet \mathbf{u}}{RT} + \frac{(\boldsymbol{\xi}_\alpha \bullet \mathbf{u})^2}{2(RT)^2} - \frac{\mathbf{u}^2}{2RT} \right\} \tag{4}$$