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Complete pivoting strategy for the left-looking Robust Incomplete Factorization preconditioner

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ABSTRACT

In this paper, we have used a complete pivoting strategy to compute the left-looking version of *RIF* preconditioner. This pivoting is based on the complete pivoting strategy of the *IJK* version of Gaussian Elimination process. There is a parameter α to control the pivoting process. To study the effect of α on the quality of the left-looking version of *RIF* preconditioner with complete pivoting strategy, we have used ten different values of this parameter. In the numerical experiments section, the quality of the left-looking version of *RIF* preconditioner with complete pivoting strategy has been compared to the quality of the right-looking version of this preconditioner.

1. Introduction

Consider the linear system of equations of the form

$$Ax = b, \tag{1}$$

where the coefficient matrix $A \in \mathbb{R}^{n\times n}$ is nonsingular, large, sparse and nonsymmetric and also $x, b \in \mathbb{R}^n$. Krylov subspace methods can be used to solve this system [1].

An implicit preconditioner M for system (1) is an approximation of matrix A, *i.e.*, $M \approx A$. If M is a good approximation of A, then it can be used as the right preconditioner for system (1). In this case, instead of solving system (1), it is better to solve the right preconditioned system

$$AM^{-1}u = b; \qquad x = M^{-1}u,$$

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by the Krylov subspace methods. *ILU* preconditioners are examples of the implicit preconditioners. This type of preconditioners are in the form of $M = LDU$ where L and U^T are unit lower triangular matrices and D is a diagonal matrix. Different versions of Gaussian Elimination process [1] can be used to compute the *ILU* preconditioners. In this case, one should work with the Schur-Complement matrices, explicitly and the computation of L and U depends on to each other.

Every matrix M which is an approximation of A^{-1}, *i.e.*, $M \approx A^{-1}$, can be considered as an explicit preconditioner of system (1). If M is a good approximation of A^{-1}, then it is recommended to solve the right preconditioned system $AMu = b$, where $x = Mu$, instead of system (1). The *AINV* preconditioner is an example of explicit preconditioners [2]. This preconditioner is computed when we apply dropping strategies in the A-biconjugation process. There are two left and right-looking versions for the A-biconjugation process. Therefore, there are two left and right-looking versions for the *AINV* preconditioner.

Robust Incomplete Factorization preconditioner or *RIF* is an example of *ILU* preconditioners [3]. This preconditioner is computed as the by-product of the *AINV* preconditioner. This means that there are also two left and right-looking versions for this preconditioner. To compute both versions of the *RIF* preconditioner, we do not need to work with the Schur-Complement matrices, explicitly and the L and U are computed, independently.

In [4], Bollhöfer and Saad proved that at the beginning of step i of the right-looking version of A-biconjugation process, one can implicitly compute the whole Schur-Complement matrix which is obtained at the end of step $i-1$ of the *IJK* version of Gaussian Elimination process. This was the main key to extend the complete pivoting strategy of the *IJK* version of Gaussian elimination process to the right-looking versions of *AINV* and *RIF* preconditioners [5,6].

Consider the ith step of the left-looking version of A-biconjugation process. At the end of this step, it is only possible to implicitly compute the first column and the first row of the Schur-Complement matrix which is obtained at the end of step $i-1$ of the *IJK* version of Gaussian Elimination process. This relation, enabled us to extend the complete pivoting strategy of the *IJK* version of Gaussian Elimination process to the left-looking version of *AINV* preconditioner [7].

In this paper, we present the complete pivoting strategy of the left-looking version of *RIF* preconditioner.

This paper is organized as follows. In Section 2, we review the left-looking version of *RIF* preconditioner and in Section 3, we present a complete pivoting strategy for this preconditioner. In Section 4, the quality of both the left and the right-looking versions of *RIF* preconditioner with complete pivoting strategy will be compared.

2. Left-looking version of *RIF* preconditioner

Suppose that matrix A has the

$$A = \bar{L}\bar{D}\bar{U}, \tag{2}$$

factorization where $\bar{L}$ and $\bar{U}^T$ are unit lower triangular matrices and $\bar{D}$ is a diagonal matrix. This factorization can be computed by Algorithm 1, which is the *IJK* version of Gaussian Elimination process [1]. In this algorithm, matrices $\bar{L}$ and $\bar{U}$ are computed column-wise and row-wise, respectively. Suppose that $S^{(0)} = A$. At the end of step $i-1$ of this algorithm, the relation

$$A = \underbrace{\begin{bmatrix} 1 & & & & & & \\ & 1 & & & & & \\ & & \ddots & & & & \\ & & & 1 & & & \\ \bar{g}_1 & \bar{g}_2 & & \bar{g}_{i-1} & 1 & & \\ & & & & & \ddots & \\ & & & & & & 1 \end{bmatrix}}_{\bar{L}} \underbrace{\begin{bmatrix} \bar{d}_{11} & & & \\ & \ddots & & \\ & & \bar{d}_{i-1,i-1} & \\ & & & (\bar{S}^{(i-1)})_{j,k\geq i} \end{bmatrix}}_{\bar{S}^{(i-1)}} \underbrace{\begin{bmatrix} 1 & & & & \bar{h}_1 & & \\ & 1 & & & \bar{h}_2 & & \\ & & \ddots & & & & \\ & & & 1 & \bar{h}_{i-1} & & \\ & & & & 1 & & \\ & & & & & \ddots & \\ & & & & & & 1 \end{bmatrix}}_{\bar{U}}, \tag{3}$$

holds where $\bar{g}_k \in \mathbb{R}^{(n-k)\times 1}$ and $\bar{h}_k \in \mathbb{R}^{1\times(n-k)}$, for $1 \leq k \leq i-1$, are the already computed columns and rows of matrices $\bar{L}$ and $\bar{U}$, respectively. The submatrix $(\bar{S}^{(i-1)})_{j,k\geq i}$ in (3) is termed the Schur-Complement matrix of step $i-1$.