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Stability analysis for Zakharov–Kuznetsov equation of weakly nonlinear ion-acoustic waves in a plasma

A.R. Seadawy *

Mathematics Department, Faculty of Science and Arts, Taibah University, Al-Ula, Saudi Arabia
Mathematics Department, Faculty of Science, Beni-Suef University, Egypt

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ABSTRACT

The Zakharov–Kuznetsov (ZK) equation is an isotropic nonlinear evolution equation, first derived for weakly nonlinear ion-acoustic waves in a strongly magnetized lossless plasma in two dimensions. In the present study, by applying the extended direct algebraic method, we found the electric field potential, electric field and magnetic field in the form of traveling wave solutions for the two-dimensional ZK equation. The solutions for the ZK equation are obtained precisely and the efficiency of the method can be demonstrated. The stability of these solutions and the movement role of the waves are analyzed by making graphs of the exact solutions.

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1. Introduction

The Zakharov–Kuznetsov (ZK) equation is a very attractive model equation for the study of vortices in geophysical flows. The ZK equation appears in many areas of physics, applied mathematics and engineering. In particular, it shows up in the area of Plasma Physics [1–3]. The ZK equation governs the behavior of weakly nonlinear ion-acoustic waves in a plasma comprised of cold ions and hot isothermal electrons in the presence of a uniform magnetic field [3–6].

The ZK equation [7] is one of two well-studied canonical two-dimensional extensions of the Korteweg–de Vries equation [8]. In contrast to the Kadomtsev–Petviashvilli (KP) equation, the ZK equation has so far never been derived in a geophysical fluid dynamics context. For a derivation of the KP equation for internal waves, see [9].

Traveling wave analysis is given in [10] for the ZK equation. Soliton solutions are derived using the improved modified extended tanh-function method [10]. One-dimensional soliton, apparently inelastic [11], periodic solutions [12] and N-soliton solutions [13] have been obtained. The auxiliary equation method and the direct Hirota bilinear method were applied to the quantum ZK equation in [14–18].

This paper is organized as follows. In Section 2, the problem formulations to derive the nonlinear two-dimensional ZK equation are presented. In Section 3, the conservation laws for the ZK equation of weakly nonlinear ion-acoustic waves in a plasma are found. In Section 4, the Hamiltonian system for the momentum and the sufficient condition for soliton solution stability are given. In Section 5, the electric field potential, electric field and magnetic field in the form of traveling wave solutions of the ZK equation are obtained and analyzed. Finally the paper ends with a conclusion in Section 6.

2. Problem formulations

Consider a low-β plasma in a magnetic field $\mathbf{B} = B\widehat{e_n}$, with $T_e \gg T_i$, where T is the temperature and the subscripts i and e denote ions and electrons respectively. The ion motions are governed by

$$\frac{\partial n}{\partial t} + \nabla \cdot (nv) = 0, \tag{1}$$

* Correspondence to: Mathematics Department, Faculty of Science, Beni-Suef University, Egypt.
E-mail address: Aly742001@yahoo.com.

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$$\frac{\partial v}{\partial t}+(v\cdot\nabla)v=-\frac{e}{m_i}\nabla\phi+v\wedge\Omega_i, \tag{2}$$

$$\nabla^2\phi=-4\pi e(n-n_e), \tag{3}$$

$$n_e=n_o\exp\left(\frac{e\phi}{kT}\right), \tag{4}$$

where n is the number density of ions, v is their velocity, m_i is the mass of an ion, ϕ is the electric field potential, and $\Omega_i=\frac{eB}{m_ic}$. Let us non-dimensionalize the various quantities as follows:

$$n'=\frac{n}{n_0},\qquad n'_e=\frac{n_e}{n_0},\qquad v'=\frac{v\sqrt{m_i}}{\sqrt{kT_e}},\qquad \phi'=\frac{\phi e}{kT_e},$$
$$x'=\frac{x}{\rho},\qquad z'=\frac{z}{L},\qquad t'=\frac{t\sqrt{m_i}}{L\sqrt{kT_e}},\qquad \rho=\frac{\sqrt{kT_e}}{\Omega_i\sqrt{m_i}}, \tag{5}$$

where L is the scale length of the waves. The independent variables are

$$\zeta=\sqrt{\epsilon}(z-t),\qquad \eta=\sqrt{\epsilon}x,\qquad \tau=\epsilon^{\frac{3}{2}}t, \tag{6}$$

where ϵ is the small parameter characterizing the typical amplitude of the waves, and using the reduction perturbation method, from Eqs. (1)–(4) the ZK equation can be derived as

$$\frac{\partial\phi}{\partial\tau}+\mu\phi\frac{\partial\phi}{\partial\zeta}+\frac{1}{2}\frac{\partial^3\phi}{\partial\zeta^3}+\frac{1}{2}(1+\delta)\frac{\partial^3\phi}{\partial\eta^2\partial\zeta}=0, \tag{7}$$

where $\delta=\frac{\lambda_D^2}{\rho^2}$, $\lambda_D^2=\frac{kT_e}{4\pi n_0e^2}$. Consider the traveling wave solutions as

$$\phi(\zeta,\eta,\tau)=\phi(\xi),\quad \text{and}\quad \xi=k\zeta+\ell\eta+\omega\tau, \tag{8}$$

where k, ℓ and ω are wave numbers and frequency. Then Eq. (7) becomes

$$\omega\phi'+k\phi\phi'+\frac{1}{2}k^3\phi^{(3)}+\frac{1}{2}k\ell^2(1+\delta)\phi^{(3)}=0. \tag{9}$$

3. Conservation laws

The ZK equation possesses some polynomial conservation laws, which can be cast into the following conservation forms:

$$\phi_\tau+\frac{1}{2}\left(\phi^2+\phi_{\zeta\zeta}+(1+\delta)\phi_{\eta\eta}\right)_\zeta=0, \tag{10}$$

$$\left(\frac{1}{2}\phi^2\right)_\tau+\frac{1}{12}\left(4\phi^3+6\phi\phi_{\zeta\zeta}-3\phi_\zeta^2+6(1+\delta)\phi\phi_{\eta\eta}+3(1+\delta)\phi_\eta^2\right)_\zeta-\frac{1}{2}(1+\delta)\left(\phi_\zeta\phi_\eta\right)_\eta=0. \tag{11}$$

Thus, if the electric field potential vanishes sufficiently rapidly at the ends of some intervals, it is easy to show that three integrals of motion (conserved quantities) exist for Eq. (7):

$$I_1[\phi]=\iint\phi\,d\zeta\,d\eta, \tag{12}$$

$$I_2[\phi]=\frac{1}{2}\iint|\phi|^2\,d\zeta\,d\eta, \tag{13}$$

$$I_3[\phi]=\frac{1}{2}\iint\left(2\phi^3+\left|\frac{\partial\phi}{\partial\zeta}\right|^2+(1+\delta)\left|\frac{\partial\phi}{\partial\eta}\right|^2\right)d\zeta\,d\eta. \tag{14}$$

These integrals of motion were also found by Infeld [11] using a variational formulation, and can provide rigorous information about the outcome of collisions of ZK solitary waves.

4. Stability analysis

Eq. (7) is a Hamiltonian system for which the momentum is given by

$$M=\frac{1}{2}\int_{-\infty}^{\infty}\phi^2d\xi \tag{15}$$

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